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SUPPLY OPTIMIZATION IN CASE OF RESOURCE RANDOM DEMAND

Abstract: The task of defining optimal resource content is formulated. Mathematical models of demand probability distribution density are considered. Optimum criteria – minimization of the mathematical expectancy of expenditure on resource excess preservation and its deficiency losses – is used. Optimum values of resource supply in case when the demand for this resource is a constant random value are set. The models the resource demand probability for which decreases with the rise of this demand and with rising distribution densities of resource demand probability are analyzed.

Key of words: supply optimization, resource demand, resource demand distribution density.

1. Introduction

Different kinds of human activity require resource supply. On the level of a company supplies belong to the objects which require considerable investments. For this reason, they are considered to be one of the factors that determine company's strategy and tactics. Supplies have always been considered an element which provides the security of distribution system and its flexible functioning.

One of the stimuli for creating supplies is the value of their negative level (deficit). In case of the deficit, there are three possible types of losses further enlisted in the order of their negative influence increase: expenses being a result of a delay in fulfilling an order; expenses conditioned by the loss of sales, when consumers make their purchasing in another company; expenses resulting from the loss of clients.

The aim of the investigation is to determine optimal values of resource supply in case when the demand for this resource is a constant value. Different mathematical models of demand probability distribution density are considered for the first time.

2. The analysis of the latest investigations and publications.

The problem of supplies functioning was investigated by Bukan J., Dudorin V., Zhdanov S., Zavelskyi M., Ivanykov Y., Kuzyn B., Fedoseev V., Ryzhykov Y. and others [1-11]. The analysis of the tasks of supplies management in mathematical aspect is fully accomplished in the book [10]. However, these works considered models with discrete distribution of probabilities. Therefore, we find it necessary that models the resource demand of which is a constant value should be taken into consideration.

3. Information layout.

We will consider economical aspect of the problem. Let the demand r for a definite type of resource during some interval of time is a constant random value with certain probability distribution density $f(r)$. If the demand does not exceed resource supply s ($r < s$), expenses caused by the resource excess stock $s - r$ are equal to c_1 per item. For $r > s$, the loses resulting from resource deficit will be equal to c_2. Optimum resource supply should be determined.

Mathematical expectation of the expenses for resource excess stock and loses from its deficit is set as a criterion of optimality. Mathematically, this criterion may be defined as follows

$$M(s) = c_1 \int_0^s (s-r) f(r)dr + c_2 \int_s^R (r-s) f(r)dr \to \min \qquad (1)$$

Here R is maximum demand ($0 \le r \le R$).

Considering calculations for definite types of the density of probability distribution, $f(r)$ will be

$$f(r) = \frac{l+1}{lR}\left(1 - \left(\frac{r}{R}\right)^l\right), \quad 0 \le r \le R, \quad l \ge 0 \qquad (2)$$

This mathematical model reflects the case when resource demand probability decreases with the increase of this demand.

Using expression (2) in formula (1) will result in:

$$M(s) = \frac{l+1}{lR}\left(c_1 \int_0^s (s-r)\left(1 - \left(\frac{r}{R}\right)^l\right) dr + c_2 \int_s^R (r-s)\left(1 - \left(\frac{r}{R}\right)^l\right) dr \right).$$

After calculating integrals:

$$M(s) = \frac{l+1}{lR}\left(c_1\left(\frac{s^2}{2} - \frac{s^{l+2}}{R^l(l+1)(l+2)}\right) + c_2\left(\frac{R^2+s^2}{2} - Rs - \frac{R^{l+2} - s^{l+2}}{R^l(l+2)} + \frac{R^{l+1}s - s^{l+2}}{R^l(l+1)}\right)\right).$$

The minimum of functional relation $M(s)$ will be investigated

$$M'(s) = \frac{l+1}{lR}\left(c_1\left(s - \frac{s^{l+1}}{R^l(l+1)}\right) + c_2\left(s - R - \frac{s^{l+1}}{R^l} + \frac{R}{(l+1)} - \frac{l+2}{l+1}\cdot\frac{s^{l+1}}{R^l}\right)\right).$$

In case when the first derivative is equal to zero ($M'(s) = 0$), the following results will be obtained:

$$\frac{c_2 + c_1}{lR}\left(s - \frac{s^{l+1}}{R^l(l+1)}\right) - \frac{1}{l+1}c_2 R = 0. \qquad (3)$$

The solution of this equation determines optimum level s_0 of resource supply. Let us analyze formula (3) for $l = 1$ and $l = 0$.

In case $l = 1$, the following result will be obtained:

$$(c_1 + c_2)s^2 - 2R(c_2 + c_1)s + R^2 c_2 = 0.$$

Its solution

$$s_0 = \frac{2R(c_1 + c_2) - \sqrt{D}}{2(c_1 + c_2)}, \text{ where } D = 4R^2 c_1 (c_1 + 2c_2).$$

And $s_0 = R\left(1 - \sqrt{\frac{c_1}{c_1 + c_2}}\right).$

Supposing

$$f(r) = \frac{l+1}{R}\left(1 - \frac{r}{R}\right)^l, \; 0 \le r \le R, \quad l \ge 0. \tag{4}$$

The implementation of formula (1) will result in:

$$M(s) = \frac{(c_2 + c_1)^R}{l+2}\left(1 - \frac{s}{R}\right)^{l+2} + c_1 s - c_1 \frac{R}{l+2}.$$

For s_0 the following results will be obtained:

$$s_0 = R\left(1 - \left(\frac{c_1}{c_1 + c_2}\right)^{\frac{1}{l+1}}\right). \tag{5}$$

In case l = 0, probability distribution will be even. Then

$$s_0 = \frac{c_2 R}{c_1 + c_2}. \tag{6}$$

Functional connection (5) may be graphically represented as follows:

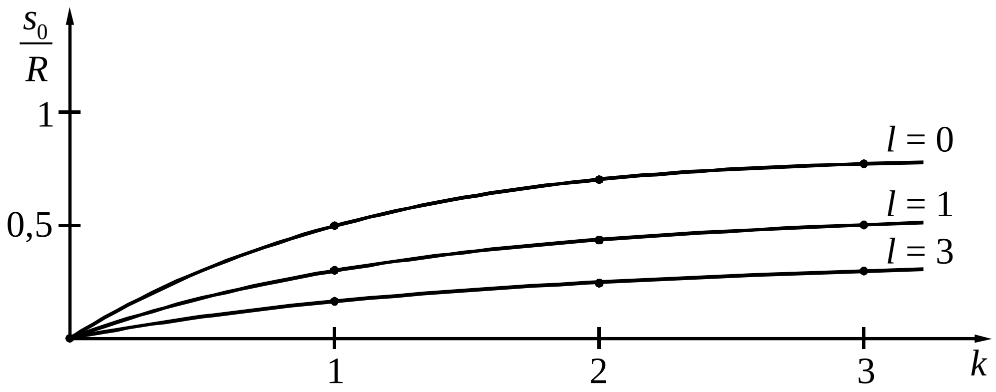

Figure 1. The dependence of optimum resource supply on the parameter $k = \frac{c_2}{c_1}$ in case of different values of *l*

Source: author's own work.

The diagram proves that value $\frac{s_0}{R}$ rises in case of the increase of *k*.

Let us investigate mathematical models with increasing resource demand probability distribution density. Supposing $f(r) = \frac{l+1}{R}\left(\frac{r}{R}\right)^l, 0 \le r \le R, \quad l \ge 0.$

The application of mathematical criterion, formula (1), in particular, will result in:

$$M(s)=\frac{l+1}{R}\left(c_1\frac{s^{l+2}}{R^l}\cdot\frac{1}{(l+1)(l+2)}+c_2\left(\frac{1}{R^l(l+2)}\left(R^{l+2}-s^{l+2}\right)-\frac{R}{l+1}\left(s-\frac{s^{l+2}}{R^{l+1}}\right)\right)\right).$$

Supposing that $M'(s)=0$, optimum value s_0 of the resource will be calculated.

$$s_0=R\left(\frac{c_2}{c_1+c_2}\right)^{\frac{1}{l+1}}. \qquad (7)$$

In case $l = 0$, formula (6) will be obtained. As a result, $M''(s_0)>0$, which is the sufficient minimum, is carried out. Formula (7) is represented graphically in the following way.

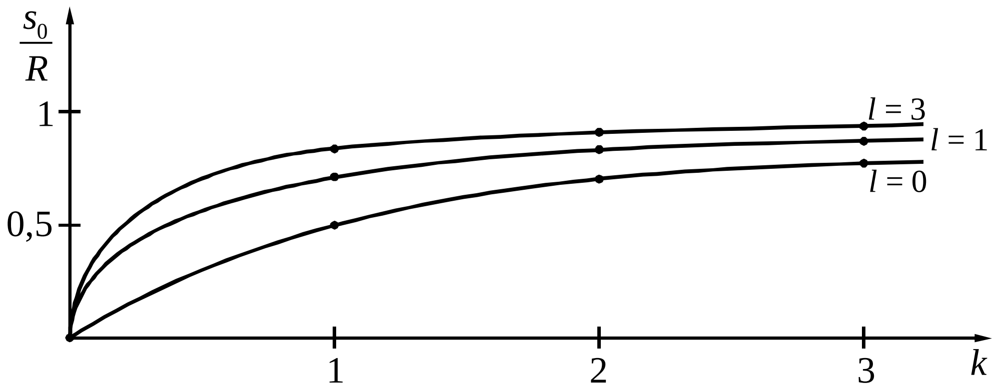

Figure 2 The dependence of optimum resource supply on the parameter $k=\frac{c_2}{c_1}$ in case of different values of *l*

Source: author's own work.

Supposing that $f(r)=\frac{l+1}{(l+2)R}\left(1+\left(\frac{r}{R}\right)^l\right)$, $0\le r\le R$, $l\ge 0$. Then

$$M(s)=\frac{l+1}{(l+2)R}\left(c_1\left(\frac{s^2}{2}+\frac{1}{(l+1)(l+2)}\cdot\frac{s^{l+2}}{R^l}\right)+c_2\left(\frac{l+4}{2(l+2)}R^2+\frac{s^2}{2}-\frac{l+2}{l+1}Rs+\frac{1}{(l+1)(l+2)}\frac{s^{l+2}}{R^l}\right)\right).$$

The equation $(c_1+c_2)s^{l+1}+(c_1+c_2)R^ls-\frac{l+2}{l+1}c_2R^{l+1}=0.$ will be used for calculating optimum value s_0.

In case $l = 0$, formula (6) will be obtained. For $l = 1$, quadratic equation $2(c_1+c_2)s^2+2(c_1+c_2)Rs-3c_2R^2=0$ will be obtained.

Its solution $\frac{s_0}{R}=\frac{-c_1-c_2+\sqrt{(c_1+c_2)(c_1+7c_2)}}{2(c_1+c_2)}$.

Calculations for $f(r)$ function will be made

$$f(r)=k\left(b-\frac{a}{R^2}\left(r-r_0\right)^2\right),\ 0\le r\le R,\ 0\le r_0\le R,\ a\ne 0,\ b>0.$$

Resulting from $\int_O^R f(r)dr=1$, parameter k will be obtained:

$$k = \frac{3R^2}{3bR^2 - a\left(R^2 - 3r_0R + 3r_0^2\right)}.$$

The following condition should be fulfilled: $3bR^2 - a\left(R^2 - 3r_0R + 3r_0^2\right) > 0.$ It will always be fulfilled in case of $a < 0$. When $a > 0$, the following correlation will be obtained

$$\begin{cases} bR^2 > ar_0^2, \\ bR^2 > a\left(R - r_0\right)^2. \end{cases}$$

In case $r = r_{0,}$ function $f(r)$ is at its maximum for $a > 0$. In case $a < 0$, $f_{min} = kb$. The following result of the criterion check calculated according to formula (1) will be obtained

$$\frac{M(s)}{k} = c_1\left(\frac{bs^2}{2} - \frac{ar_0^3}{3R^2}s - \frac{a}{12R^2}\left(s - r_0\right)^4 + \frac{a}{12R^2}r_0^{\,4}\right) +$$

$$+c_2\left(\frac{bs^2}{2} - \frac{a}{12R^2}\left(s - r_0\right)^4 + \frac{as}{3R^2}\left(R - r_0\right)^3 - bRs + \frac{bR^2}{2} - \frac{a}{3R}\left(R - r_0\right)^3 + \frac{a}{12R^2}\left(R - r_0\right)^4\right).$$

If $M'(s) = 0$, the following equation for calculating the value s_0 of resource supply will be obtained

$$\left(c_1 + c_2\right)a\left(s - r_0\right)^3 - 3R^2\left(c_1 + c_2\right)bs - c_2a\left(R - r_0\right)^3 + 3R^3c_2b = 0.$$

Let us consider mathematical models for unlimited demand. Probability distribution density $f(r)$ will be selected due to $f(r) = \frac{\left(l - 1\right)a^{l-1}}{\left(r + a\right)^l}$, $l \geq 3$, $a > 0$, $r \geq 0$.

Formula for $M(s)$ in this case will be as follows

$$M(s) = c_1\int_0^s (s - r) f(r) dr + c_2\int_s^\infty (r - s) f(r) dr. \tag{8}$$

After the substitution of a definite function $f(r)$ and integrating, the results will be as follows

$$M(s) = c_1\left(s - \frac{3a^{l-1}}{(l + 2)(s + a)^{l-2}} + \frac{a(2l - 3)}{(l - 2)}\right) + c_2\frac{a^{l-1}}{(l - 2)(s + a)^{l-2}}.$$

Optimum value of resource supply in this case will be calculated according to formula

$$s_0 = \left(\left(\frac{c_1 + c_2}{c_1}\right)^{\frac{1}{l}} - 1\right)a. \tag{9}$$

Dependency diagram (9) will be:

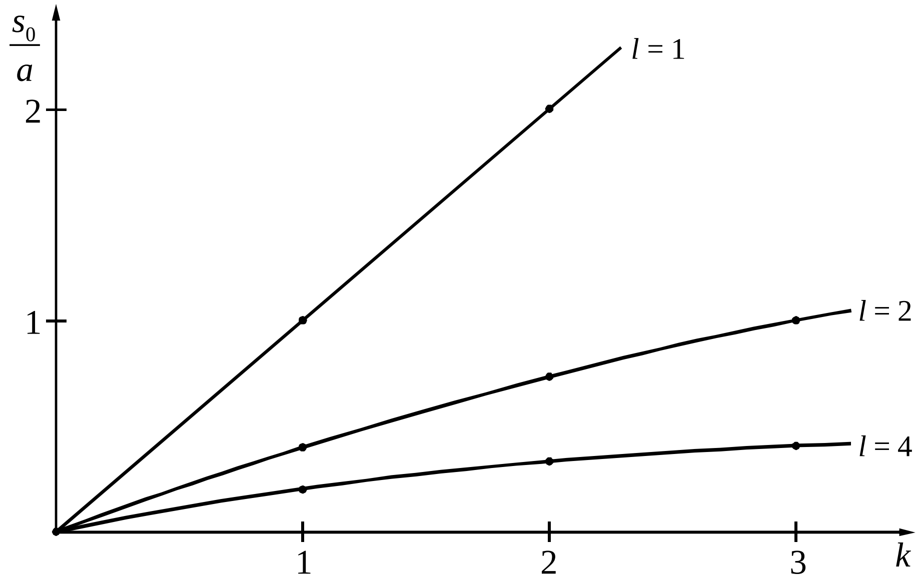

Figure 3 The dependence of optimum resource supply on the parameter $k = \frac{c_2}{c_1}$

Source: author's own work.

Supposing $f(r) = \frac{r}{a^2} e^{-\frac{r}{a}}$, $a > 0$, $r \geq 0$. $M(s)$ will be calculated according to formula (8).

$$M(s) = c_1\left(s + se^{-\frac{s}{a}} - 2a + 2ae^{-\frac{s}{a}}\right) + c_2\left(se^{-\frac{s}{a}} + 2ae^{-\frac{s}{a}}\right),$$

$M'(s)$ is to be calculated.

$$M'(s) = c_1 - \frac{c_1 s}{a} e^{-\frac{s}{a}} - c_1 e^{-\frac{s}{a}} - c_2 e^{-\frac{s}{a}} - \frac{c_2 s}{a} e^{-\frac{s}{a}}.$$

Provided $M'(s) = 0$, the following equation for calculating s_0 value will be obtained

$$c_1 a - e^{-\frac{s}{a}}(s + a)(c_1 + c_2) = 0.$$

4.Conclusions

Further, practical significance of the obtained results will be grounded.

Supposing that statistical distribution of resource demand looks as follows

$r_i - r_{i+1}$	0–2	2–4	4–6	6–8	8–10
n_i	14	3	1	1	1

Formula (4) will be used for mathematical modeling of this distribution. Supposing R = 10, the parameter l will be obtained with the help of point method, in case mathematical expectation is equal to the average ($M(r) = \bar{r}$). For this statistical distribution $\bar{r} = 2,2.$ Having calculated mathematical expectation M(r) of possibilities distribution (4), the following result will be obtained: $M(r) = \frac{R}{l+2}.$ Parameter l is as follows l = 2,54. Optimum value s0 of resource supply will be calculated according to formula (5), on condition that c2 = 2c1. So, the resource supply is s0 = 2,67.

Other optimum criteria will be considered in further investigations.

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ОПТИМІЗАЦІЯ ЗАПАСІВ ПРИ ВИПАДКОВОМУ ПОПИТІ НА РЕСУРС

Анотація: Сформульовано задачу визначення оптимального запасу ресурсу. Розглянуто різні математичні моделі густини розподілу імовірності попиту. Використано критерій оптимальності — мінімізація математичного сподівання витрат на зберігання надлишку ресурсу та втрат від його дефіциту. Визначено оптимальні величини запасу ресурсу у випадку, коли попит на цей ресурс є неперервною випадковою величиною. Проаналізовані моделі, для яких ймовірність попиту на ресурс зменшується із зростанням цього попиту і із зростаючими густинами розподілу ймовірностей попиту на ресурс.

Ключові слова: оптимізація запасу, попит на ресурс, густина розподілу ймовірностей попиту.