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Robust Stability of Linear Control System with Matrix Uncertainty

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Resume. *The work is devoted to working out of new methods for analysis of robust stability and robust stabilization of linear dynamic systems. Sufficient stability conditions of the zero state are formulated for a linear system with uncertain coefficient matrices and a measured output feedback. In addition, a common quadratic Lyapunov function and ellipsoidal set of stabilizing matrixes of amplification factors of a output feedback are given for the whole set of system. Application of the results is reduced to a solution of systems of linear matrix inequalities.*

Keywords: *control system, output feedback, robust stability, matrix uncertainty, ellipsoid.*

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Problem setting. In the applied problems of analysis and synthesis of real objects differential and difference systems with uncertain parameters and functional structure are used (see, eg, [1] - [4]). This focuses on the objectives of robust stability and robust stabilization.

As set robust stability of dynamic systems we mean parametric or functional set characterizing uncertainty of the given structure of the system and its controlling elements. In particular, in the uncertain linear models matrix of coefficients and feedback may belong to some given sets in the corresponding spaces (polytopes, ellipsoids, matrix spacing, etc.).

The task of stabilizing of the control system is to build a static or dynamic control to ensure the asymptotic stability of the equilibrium of the closed-loop system with arbitrary values of uncertain elements. Typically, this problem is reduced to solving systems of linear matrix inequalities (LMI).

Analysis of recent research and publications. To describe the uncertainties and conditions of robust stability of systems matrix intervals and polytopes are used

[1, 5, 6]. In the works [3, 7] in terms of linear matrix inequalities sufficient conditions for the stability of linear control systems with uncertain coefficient matrix and feedback have been obtained. One can find the view of problems and known methods of robust stability analysis and stabilization of control systems with feedback in [8, 9].

The aim of the research is to develop new methods of analysis of robust stability and robust stabilization of linear dynamic systems with limited at a rate matrix uncertainties and static measurable output feedback.

Robust stabilization of control systems. We will consider the continuous linear dynamical control system:

$$\dot{x} = (A + \Delta A(t))x + (B + \Delta B(t))u, \quad u = Ky, \quad y = Cx + Du, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^l$ are respectively the state, control, and observable object output vectors, A, B, C and D are constant matrices of the corresponding sizes $n \times n$, $n \times m$, $l \times n$ i $l \times m$, and

$$\Delta A(t) = F_A \Delta_A(t) H_A, \quad \Delta B(t) = F_B \Delta_B(t) H_B, \tag{2}$$

where F_A, F_B, H_A, H_B are constant matrices of corresponding size and matrix uncertainties $\Delta_A(t)$ i $\Delta_B(t)$ satisfy the constraints

$$\|\Delta_A(t)\| \le 1, \; \|\Delta_B(t)\| \le 1 \text{ or } \|\Delta_A(t)\|_F \le 1, \; \|\Delta_B(t)\|_F \le 1, \quad t \ge 0. \tag{3}$$

Hereinafter, $\|\cdot\|$ – Euclidean vector norm and spectral matrix norm, $\|\cdot\|_F$ – matrix Frobenius norm, I – unit matrix of corresponding size. To simplify the records of the matrices dependency on t we will omit.

It should be noted that when $\Delta_A(t) = 0$, $\Delta_B(t) = 0$ the system changes into the system without uncertainties under review [10].

We will formulate known criteria of positive and nonnegative definiteness of block matrices.

Lemma 1. [11] *There is an equivalence:*

$$\begin{bmatrix} U & Z \\ Z^T & V \end{bmatrix} > 0 \Leftrightarrow V > 0, \quad U - ZV^{-1}Z^T > 0. \tag{4}$$

If the block V is nondegenerate, then

$$\begin{bmatrix} U & Z \\ Z^T & V \end{bmatrix} \geq 0 \Leftrightarrow V > 0, \quad U - ZV^{-1}Z^T \geq 0. \tag{5}$$

Lemma 2. [12] *Suppose that the following system of matrix inequalities hold:*

$$\begin{bmatrix} R - P^{-1} & D^T \\ D & -Q^{-1} \end{bmatrix} < 0, \quad \begin{bmatrix} W & U^T & V^T \\ U & R - P^{-1} & D^T \\ V & D & -Q^{-1} \end{bmatrix} \leq 0 \; (< 0), \tag{6}$$

where $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T \geq 0$, $W = W^T \geq 0$, U, V *and* D *are matrices of* corresponding *sizes. Then for any matrix* $K \in \mathcal{E}$ *the matrix inequality*

$$W + U^T \mathcal{D}(K) V + V^T \mathcal{D}^T(K) U + V^T \mathcal{D}^T(K) R \mathcal{D}(K) V \leq 0 \; (< 0) \tag{7}$$

is implemented.

Lemma 3. [13] *Suppose that* L *is symmetric matrix, the matrix* $M_1, \ldots, M_r$ *and* $N_1, \ldots, N_r$ *have corresponding dimensions. Then, if for some numbers* $\varepsilon_1, \ldots, \varepsilon_r > 0$ *matrix inequality*

$$L + \sum_{i=1}^{r} \left(\varepsilon_i M_i M_i^T + \frac{1}{\varepsilon_i} N_i^T N_i \right) \leq 0$$

is performed, then the inequality

$$L + \sum_{i=1}^{r} \left(M_i \Delta_i N_i + (M_i \Delta_i N_i)^T \right) \leq 0$$

is true for all $\|\Delta_i\| \leq 1$ *or* $\|\Delta_i\|_F \leq 1$, $i = 1, \ldots, r$.

We will note that Lemmas 2 and 3 are generalizations of the sufficiency statement of the adequacy criterion called Petersen's lemma on matrix uncertainty [14].

The set of control matrices K providing stability in the closed-loop system we will build as an ellipsoid

$$\mathcal{E} = \left\{ K \in \mathbb{R}^{m \times l} : K^T P^{-1} K \leq Q \right\}, \tag{8}$$

where $P = P^T > 0$ and $Q = Q^T > 0$ are some positively defined matrices.

We introduce on the set of matrices $\mathcal{K} = \{ K : \det(I_m - KD) \neq 0 \}$ a nonlinear operator

$$\mathcal{D} : \mathbb{R}^{m \times l} \to \mathbb{R}^{m \times l}, \quad \mathcal{D}(K) = (I_m - KD)^{-1} K \equiv K (I_l - DK)^{-1}.$$

For the operator $\mathcal{D}$ the property is performed [12]: if $K_1 \in \mathcal{K}$ and $K_2 \in \mathcal{K}$ than

$$K_3 = (I_m - K_1 D)^{-1} K_2 \in \mathcal{K}, \quad \mathcal{D}(K_1 + K_2) \equiv \mathcal{D}(K_1) + \mathcal{D}(K_3) \left[I_l + D \mathcal{D}(K_1) \right]. \tag{9}$$

With (1) and (8) follows the inequality

$$w_0(x,u) = \left[x^T, u^T\right]\begin{bmatrix} C^T QC & C^T QD \\ D^T QC & D^T QD - P^{-1}\end{bmatrix}\begin{bmatrix} x \\ u\end{bmatrix} \geq 0 . \qquad (10)$$

Supposed

$$D^T QD < P^{-1} . \qquad (11)$$

This is equivalent to the first block inequality of Lemma 2 with $R = 0$. Then $x = 0$ implies $u = 0$, and $x \equiv 0$ is an equilibrium state for the system whose stability we are examining. The closed-loop system has the following structure

$$\dot{x} = M(t)x, \quad M(t) = A + \Delta A + (B + \Delta B)\mathcal{D}(K)C . \qquad (12)$$

In conditions (8) and (9) we have $D^T K^T P^{-1} KD \leq D^T QD < P^{-1}$. According to Lyapunov's theorem for discrete systems $\rho(KD) < 1$, and therefore in (12) $I_m - KD$ is a nondegenerate matrix.

We will describe the conditions of robust stabilization of the class systems (1).

Theorem 1. *Suppose that for the Hurwitz matrix* A *of the system (1) and for some* $\varepsilon_1, \varepsilon_2 > 0$ *the following matrix inequalities hold: (11) and*

$$\begin{bmatrix} A^T X + XA + \varepsilon_1 H_A^T H_A & XB & C^T & XF_A & XF_B \\ B^T X & -P^{-1} + \varepsilon_2 H_B^T H_B & D^T & 0 & 0 \\ C & D & -Q^{-1} & 0 & 0 \\ F_A^T X & 0 & 0 & -\varepsilon_1 I & 0 \\ F_B^T X & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} \leq 0, \qquad (13)$$

where $X = X^T > 0$. *Then control* $u = Ky$ *with arbitrary matrix* $K \in \mathcal{E}$ *stabilizes the system (1). Moreover, if in (13) holds strict matrix inequality, the given set of controls ensures asymptotic stability of the closed-loop system (12) and a common Lyapunov's function* $v(x) = x^T Xx$.

Proff. We construct Lyapunov's function for the closed-loop system (12) as $v(x) = x^T Xx$. Then by Lyapunov's Theorem system (12) is stable (asymptotically stable) if for some positive definite matrix $X = X^T > 0$ matrix inequality

$$(A + \Delta A + (B + \Delta B)\mathcal{D}(K)C)^T X + X(A + \Delta A + (B + \Delta B)\mathcal{D}(K)C) \leq 0 \ (< 0) \qquad (14)$$

is implemented. We rewrite the last inequality in the form

$$(A + \Delta A)^T X + X(A + \Delta A) + C^T \mathcal{D}^T(K)(B + \Delta B)^T X + X(B + \Delta B)\mathcal{D}(K)C \leq 0$$

and use lemma 2 putting

$$U=(B+\Delta B)^T X\,,\;\; V=C\,,\;\; W=(A+\Delta A)^T X+X(A+\Delta A)\,,\;\; R=0\,.$$

Then the second block inequality in (6) has the form

$$\begin{bmatrix} (A+\Delta A)^T X+X(A+\Delta A) & X(B+\Delta B) & C^T \\ (B+\Delta B)^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} \le 0\,. \tag{15}$$

Using the structure of matrix uncertainties $\Delta_A(t)$, $\Delta_B(t)$, we decompose the last inequality

$$\begin{bmatrix} A^T X+XA & XB & C^T \\ B^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} + \begin{bmatrix} H_A^T \\ 0 \\ 0 \end{bmatrix} \Delta_A^T \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \Delta_A \begin{bmatrix} H_A & 0 & 0 \end{bmatrix} +$$

$$+\begin{bmatrix} 0 \\ H_B^T \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} 0 & H_B & 0 \end{bmatrix} \le 0\,,$$

which is done for lemma 3 if there are $\varepsilon_1,\varepsilon_2>0$ such as

$$\begin{bmatrix} A^T X+XA & XB & C^T \\ B^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} + \varepsilon_1 \begin{bmatrix} H_A^T H_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_1} \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$

$$+\varepsilon_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & H_B^T H_B & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_2} \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} \le 0\,.$$

According to lemma 1 the obtained matrix inequality is equivalent to inequality (13).

The theorem is proved.

We can give another proof of Theorem 1 using the theorem of S- procedure for quadratic forms with one restriction [15]. It argues that inequality $w(x,u)\le 0 \;\; (<0)$ with limit $w_0(x,u)\ge 0$ is equivalent to the ratio

$$w(x,u)+\tau w_0(x,u)\le 0\;(<0)\,,\;\; x^T x+u^T u\ne 0\,, \tag{16}$$

where $\tau>0$ – a certain number. We can give $\tau=1$. Then according to (10) and the derivative of the system

$$w(x,u)=\begin{bmatrix} x^T, u^T \end{bmatrix} \begin{bmatrix} (A+\Delta A)^T X+X(A+\Delta A) & X(B+\Delta B) \\ (B+\Delta B)^T X & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \le 0\;(<0)\,,$$

we rewrite (16) as

$$[x^T, u^T]\begin{bmatrix} \Omega & X(B+\Delta B)+C^T QD \\ (B+\Delta B)^T X + D^T QC & D^T QD - P^{-1} \end{bmatrix}\begin{bmatrix} x \\ u \end{bmatrix} \le 0\ (<0),$$

where $\Omega = (A+\Delta A)^T X + X(A+\Delta A) + C^T QC$. Using lemma 1, we obtain the inequality (15).

In theorem 1 the system (1) without controlling ($u=0$) should be stable. If the zero state of the system (1) without control is unstable, then we will look for a set of stabilizing controls from the ellipsoid

$$\mathcal{E}_0 = \{K \in \mathbb{R}^{m\times l} : (K-K_0)^T P^{-1}(K-K_0) \le Q\},$$

which is equivalent to the matrix choice

$$K = K_0 + \widetilde{K}, \quad \widetilde{K} \in \mathcal{E}. \tag{17}$$

Firstly we have to obtain matrix K_0 which stabilizes the system

$$\dot{x} = M_0 x, \quad M_0 = A + \Delta A + (B+\Delta B)\mathcal{D}(K_0)C.$$

Matrix K_0 can be obtained with methods described in [5].

We construct conditions of robust stabilization of class (1) with the control matrix (17). According to (1), (8) and (17) the inequality

$$[x^T, u^T]\begin{bmatrix} C^T QC - C^T K_0^T P^{-1} K_0 C & C^T QD + C^T K_0^T P^{-1} G \\ D^T QC + G^T P^{-1} K_0 C & \Delta \end{bmatrix}\begin{bmatrix} x \\ u \end{bmatrix} \ge 0$$

should be performed, where $\Delta = D^T QD - G^T P^{-1} G$, $G = I_m - K_0 D$. We suppose that

$$\Delta < 0. \tag{18}$$

Then $x=0$ implies $u=0$, and $x \equiv 0$ is an equilibrium state for the system.

Under assumption (18) matrix G must be nondegenerate. Therefore values of the operator $\mathcal{D}(K_0) = (I_m - K_0 D)^{-1} K_0$ are defined. If $\widetilde{K} \in \mathcal{E}$ then values of $\mathcal{D}(K)$ and $\mathcal{D}(\hat{K})$ are also defined, where $\hat{K} = G^{-1}\widetilde{K}$. Indeed, under conditions (17) and (18) we obtain

$$D^T \widetilde{K}^T P^{-1} \widetilde{K} D \le D^T QD < G^T P^{-1} G, \quad F^T P^{-1} F < P^{-1},$$

where $F = \widetilde{K} D G^{-1}$ and $P > 0$. So $\rho(F) < 1$ and matrix $I_m - F$ is nondegenerate and nondegenerate are matrices $I_m - KD = (I_m - F)G$ and $I_m - \hat{K}D = G^{-1}(I_m - KD)$.

So, the closed-loop system (1), (17) under constraint (18) can be represented in the form (12).

Theorem 2. *Suppose that for a positive definite matrix* $X = X^T > 0$ *and some* $\varepsilon_1, \varepsilon_2 > 0$ *the following matrix inequalities hold: (18) and*

$$\begin{bmatrix} Z & XB+\varepsilon_2 C^T \mathcal{D}^T(K_0) H_B^T H_B & C_*^T & XF_A & XF_B \\ B^T X + \varepsilon_2 H_B^T H_B \mathcal{D}(K_0) C & -G^T P^{-1} G + \varepsilon_2 H_B^T H_B & D^T & 0 & 0 \\ C_* & D & -Q^{-1} & 0 & 0 \\ F_A^T X & 0 & 0 & -\varepsilon_1 I & 0 \\ F_B^T X & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} \le 0, \quad (19)$$

where $Z = (A + B\mathcal{D}(K_0)C)^T X + X(A + B\mathcal{D}(K_0)C) + \varepsilon_1 H_A^T H_A + \varepsilon_2 C^T \mathcal{D}^T(K_0) H_B^T H_B \mathcal{D}(K_0) C$.

Then control $u = Ky$ *with random matrix (17) stabilizes the system (1). Moreover, if in (19) holds strict matrix inequality, the given set of controls ensures asymptotic stability of the closed-loop system (12) and a common Lyapunov function* $v(x) = x^T X x$.

Proof. We construct Lyapunov function for the closed-loop system (12) in the form of $v(x) = x^T X x$. Stability (asymptotic stability) of the zero state equilibrium ensures matrix inequality $X = X^T > 0$ and nonpositive (negative) definite of the derivative of the system $\dot{v}(x) = w(x,u)$, ie taking into account (17) performance of matrix inequalities would be sufficient:

$$(A + \Delta A + (B + \Delta B)\mathcal{D}(K_0 + \widetilde{K})C)^T X + X(A + \Delta A + (B + \Delta B)\mathcal{D}(K_0 + \widetilde{K})C) \le 0 \ (< 0). \quad (20)$$

Applying the property (9) of the operator $\mathcal{D}(K) = (I_m - KD)^{-1}K$, rewrite inequality (20) in the form

$$(A + \Delta A)^T X + X(A + \Delta A) + C^T \left(\mathcal{D}^T(K_0) + (I + \mathcal{D}^T(K_0)D^T)\mathcal{D}^T(\hat{K})\right)(B + \Delta B)^T X +$$
$$+ X(B + \Delta B)\left(\mathcal{D}(K_0) + \mathcal{D}(\hat{K})(I + D\mathcal{D}(K_0))\right)C \le 0$$

Last inequality we rewrite as

$$M_*^T X + XM_* + C_*^T \mathcal{D}^T(\hat{K})(B + \Delta B)^T X + X(B + \Delta B)\mathcal{D}(\hat{K})C_* \le 0,$$

where $M_* = A + \Delta A + (B + \Delta B)\mathcal{D}(K_0)C$, $C_* = C + D\mathcal{D}(K_0)C$, $\hat{K} = G^{-1}\widetilde{K}$. Here

$$\widetilde{K} \in \mathcal{E} \Leftrightarrow \hat{K} \in \hat{\mathcal{E}} = \left\{K : K^T \hat{P} K \le Q\right\},$$

where $\hat{P} = G^T P^{-1} G$.

We use lemma 2 putting

$$W = M_*^T X + XM_*, \; U = (B + \Delta B)^T X, \; V = C_*, \; R = 0.$$

Then the second block inequality in (6) has the form

$$\begin{bmatrix} M_*^T X + XM_* & X(B+\Delta B) & C_*^T \\ (B+\Delta B)^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} \le 0 .$$

Using the structure of the matrix uncertainties $\Delta_A(t)$, $\Delta_B(t)$, we decompose the last inequality

$$\begin{bmatrix} A^T X + XA + C^T \mathcal{D}^T (K_0) B^T X + XB\mathcal{D}(K_0)C & XB & C_*^T \\ B^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} + \begin{bmatrix} H_A^T \\ 0 \\ 0 \end{bmatrix} \Delta_A^T \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \Delta_A \begin{bmatrix} H_A & 0 & 0 \end{bmatrix} + \begin{bmatrix} C^T \mathcal{D}^T (K_0) H_B^T \\ 0 \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} H_B \mathcal{D}(K_0) C & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ H_B^T \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} 0 & H_B & 0 \end{bmatrix} \le 0 ,$$

which according to lemma 3 is done if there are $\varepsilon_1, \varepsilon_2 > 0$ such as

$$\begin{bmatrix} A^T X + XA + C^T \mathcal{D}^T (K_0) B^T X + XB\mathcal{D}(K_0)C & XB & C_*^T \\ B^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} +$$

$$+ \varepsilon_1 \begin{bmatrix} H_A^T H_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_1} \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$

$$+ \varepsilon_2 \begin{bmatrix} C^T \mathcal{D}^T (K_0) H_B^T H_B \mathcal{D}(K_0) C & C^T \mathcal{D}^T (K_0) H_B^T H_B & 0 \\ H_B^T H_B \mathcal{D}(K_0) C & H_B^T H_B & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_2} \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} \le 0 .$$

We apply Lemma 1 and get the conditions (18) and (19), under which the inequality (20) hold for any matrix $\widetilde{K} \in \mathcal{E}$. These conditions ensure asymptotic stability for the zero state of the closed-loop system (12).

The theorem is proved.

The results of theorems 1-2 can be generalized in case when

$$\Delta A(t) = \sum_{i=1}^{r} F_A^{(i)} \Delta^{(i)}(t) H_A^{(i)} , \quad \Delta B(t) = \sum_{i=1}^{r} F_B^{(i)} \Delta^{(i)}(t) H_B^{(i)} .$$

Conclusions. In this work, new methods of analysis of robust stability of equilibrium states of control with output feedback been obtained. The values of matrix coefficients are set by restrictions on normal matrix uncertainties and dimensional vector output includes components of the system as state as control.

Feasibility of the obtained methods is reduced to solving algebraic LMI. A distinctive feature of obtained LMI from known ones is the possibility of building an ellipsoid matrix stabilizing coefficients to stimulate feedback and common quadratic Lyapunov function.

The results are obtained based on the known generalizations statement on adequacy of Petersen's lemma about matrix uncertainties. Unfortunately, the conditions of theorem 1-2 are generally theoretical. Their practical use in problems of output robust stabilization based on quadratic Lyapunov functions with uncertain matrices requires special methods of matrix K_0 (see, e.g., [5, 8]). This is one of the topical tasks of the following studies.

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Робастна стійкість лінійних керованих систем з матричними невизначеностями

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Резюме. *Робота присвячена розробленні нових методів аналізу робастної стійкості та робастної стабілізації лінійних динамічних систем. Для лінійних керованих систем з невизначеними матричними коефіцієнтами та зворотного зв'язку по вимірюваному виходу формулюються достатні умови стійкості нульового стану рівноваги. При цьому визначаються спільна квадратична функція Ляпунова та еліпсоїдальна множина стабілізуючих матриць коефіцієнтів підсилення зворотного зв'язку для всієї сім'ї систем. Практична реалізація отриманих методів зводиться до розв'язуванням систем лінійних матричних нерівностей.*

Ключові слова: *система керування, зворотній зв'язок, робастна стійкість, матрична невизначеність, еліпсоїд.*

Отримано