MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
WEST UKRAINIAN NATIONAL UNIVERSITY

Methodical instructions for solution complex practical individual tasks (CPIT) in discipline «Higher Mathematics»

Ternopil – 2022

UDK 519.2

Reviewers:

S.V. Martyniuk – PhD, Candidate of physical and mathematical sciences, associate professor of the Department of Mathematics and Teaching Methods at Volodymyr Hnatyuk Ternopil National Pedagogical University.

O.S. Bashutska – PhD, Candidate of Economics, associate Professor of the Department of Economic Cybernetics and Informatics of the West Ukrainian National University;

approved at the meeting of the Department of Applied Mathematics, protocol № 1 of 26.08.2022.

Methodical instructions for solution complex practical individual tasks (CPIT) in discipline «Higher Mathematics» include complex practical individual tasks for students and examples of their solutions.

.

Plaskon S.A., Dzyubanovska N.V. Methodical instructions for solution complex practical individual tasks (CPIT) in discipline «Higher Mathematics» . - Ternopil: WUNU, 2022.- 24 p.

UDK 519.2

Responsible for release: O.M. Martyniuk, PhD, Candidate of physical and mathematical sciences, associate professor of the Department of Applied Mathematics, Head of the Department of Applied Mathematics WUNU

Variants of tasks for realization of the complex practical practical task

To complete a complex practical practical assignment, the student chooses an option according to its number in the group list, executes and protects all tasks in accordance with the schedule.

When solving tasks, consider that the parameter is set by the teacher.

Variant	**Deadlines for the CIPIP (1st semester)**			
	Until the implementation of module1		**Until the implementation of module2**	
1	1.1., 5.1., 6.1., 7.1.	8.1., 11.1., 12.1., 13.1.	14.1., 23.1., 24.1.	17.1., 25.1., 26.1., 27.1., 28.1., 29.1., 30.1.
2	1.2., 5.2., 6.2., 7.2.	8.2., 11.2., 12.2., 13.2. а)	14.2., 23.2., 24.2.	17.2., 25.2., 26.2., 27.2., 28.2., 29.2., 30.2.
3	1.3., 5.3., 6.3., 7.3.	8.3., 11.3., 12.3., 13.2. б)	14.3., 23.3., 24.3.	17.3., 25.3., 26.3., 27.3., 28.3., 29.3., 30.3.
4	1.4., 5.4., 6.4., 7.4.	8.4., 11.4., 12.4., 13.3.	14.4., 23.4., 24.4.	17.4., 25.4., 26.4., 27.4., 28.4., 29.4., 30.4.
5	2.1., 5.5., 6.5., 7.5.	8.5., 11.5., 12.5., 13.4. а)	15.1., 23.5., 24.5.а)	17.5., 25.5., 26.5., 27.5., 28.5., 29.5., 30.5.
6	2.2., 5.6., 6.6., 7.6.	8.6., 11.6., 12.6., 13.4. б)	15.2., 23.6., 24.5.б)	17.6., 25.6., 26.6., 27.6., 28.6., 29.6., 30.6.
7	2.3., 5.7., 6.7., 7.7.	8.7., 11.7., 12.7., 13.4. в)	15.3., 23.7., 24.6.	18.1., 25.7., 26.7., 27.7., 28.7., 29.7., 30.7.
8	2.4., 5.8., 6.8., 7.8.	9.1., 11.8., 12.8., 13.5.	15.4., 23.8., 24.7.	18.2., 25.8., 26.8., 27.8., 28.8., 29.8., 30.8.
9	3.1., 5.9., 6.9., 7.9.	9.2., 11.9., 12.9., 13.6.	15.5., 23.9., 24.8.	18.3., 25.9., 26.9., 27.9., 28.9., 29.9., 30.9.
10	3.2., 5.10., 6.10., 7.10.	9.3., 11.10., 12.10., 13.7.	15.6., 23.10., 24.9.	18.4., 25.10., 26.10., 27.10., 28.10., 29.10., 30.10.
11	3.3., 5.11., 6.11., 7.11.	9.4., 11.11., 12.11., 13.8.	16.1., 23.11., 24.10.	18.5., 25.11., 26.11., 27.11., 28.11., 29.11., 30.11.
12	3.4., 5.12., 6.12., 7.12.	9.5., 11.12., 12.12., 13.9. а)	16.2., 23.12., 24.11. а)	18.6., 25.12., 26.12., 27.12., 28.12., 29.12., 30.12.
13	3.7., 5.13., 6.13., 7.13.	9.6., 11.13., 12.13., 13.9 б)	16.3., 23.13., 24.11.б)	19.1., 25.13., 26.13., 27.13.,

				28.13., 29.13., 30.13.
14	3.8., 5.14., 6.14., 7.14.	10.1., 11.14., 12.14., 13.10.	16.4., 23.14., 24.11. в)	19.2., 25.14., 26.14., 27.14., 28.14., 29.14., 30.14.
15	3.7., 5.15., 6.15., 7.15.	10.2., 11.15., 12.15., 13.11.	16.5., 23.15., 24.11. г)	19.3., 25.15., 26.15., 27.15., 28.15., 29.15., 30.15.
16	3.8., 5.16., 6.16., 7.16.	10.3., 11.16., 12.16., 13.12.	16.6., 23.16., 24.11. д)	19.4., 25.16., 26.16., 27.16., 28.16., 29.16., 30.16.
17	3.9., 5.17., 6.17., 7.17.	10.4., 11.17., 12.17., 13.13.	16.7., 23.17., 24.12. а)	19.5., 25.17., 26.17., 27.17., 28.17., 29.17.а), 30.17.
18	1.4., 5.18., 6.18., 7.18.	8.1., 11.18., 12.18., 13.14.	14.1., 23.18., 24.12. б)	20.1., 25.18., 26.18., 27.18., 28.18., 29.17.б), 30.18.
19	2.1., 5.19., 6.19., 7.19.	8.2., 11.19., 12.19., 13.15.	14.2., 23.19., 24.12. в)	20.2., 25.19., 26.19., 27.19., 28.19., 29.17.в), 30.19.
20	2.2., 5.20., 6.20., 7.20.	8.3., 11.20., 12.20., 13.1.	14.3., 23.20., 24.12. г)	20.3., 25.20., 26.20., 27.20., 28.20., 29.17.г), 30.20.
21	2.3., 5.21., 6.21., 7.21.	8.4., 11.21., 12.21., 13.2. а)	14.4., 23.21., 24.12.д)	20.4., 25.21., 26.21., 27.21. 28.21., 29.18., 21.
22	2.4., 5.22., 6.22., 7.22.	8.5., 11.22., 12.22., 13.3.	15.1., 23.22., 24.13. а)	21.1., 25.22., 26.22., 27.22., 28.22., 29.19., 30.22.
23	3.1., 5.23., 6.23., 7.23.	8.6., 11.23., 12.23., 13.4. а)	15.2., 23.23., 24.13. б)	21.2., 25.23., 26.23., 27.23., 28.23., 29.20., 30.23.
24	3.2., 5.24., 6.24., 7.24.	8.7., 11.24., 12.24., 13.5.	15.3., 23.24., 24.13.в)	21.3., 25.24., 26.24., 27.24., 28.24., 29.21.в), 30.24.
25	3.3., 5.25., 6.25., 7.25.	9.1., 11.25., 12.25., 13.6.	15.4., 23.25., 24.13. г)	21.4., 25.25., 26.25., 27.25., 28.25., 29.21.а), 30.25.
26	3.4., 5.26., 6.26., 7.26.	9.2., 11.26., 12.26., 13.7.	15.5., 23.26., 24.13. д)	22.1., 25.26., 26.26., 27.26., 28.26., 29.21.б), 30.26.

27	3.5., 5.27., 6.27., 7.27.	9.3., 11.27., 12.27., 13.8.	15.6., 23.27., 24.1.	22.2., 25.27., 26.27., 27.1., 28.27., 29.22., 30.27.
28	3.6., 5.28., 6.28., 7.28.	9.4., 11.28., 12.28., 13.9. а)	16.1., 23.28., 24.2.	22.3., 25.28., 26.28., 27.2., 28.28., 29.23., 30.28.
29	1.1., 5.29., 6.29., 7.29.	9.5., 11.29., 12.29., 13.10. б)	16.2., 23.29., 24.3.	22.4., 25.29., 26.29., 27.3., 28.29., 29.24., 30.29.
30	1.2.,5.30., 6.30., 7.30.	9.6., 11.30., 12.30., 13.10.	16.3., 23.30., 24.4.	17.1., 25.30., 26.30., 27.4., 28.30., 29.25., 30.30.
31	1.3., 5.31., 6.31., 7.31.	10.1., 11.31., 12.31., 13.11.	16.4., 23.31., 24.5. а)	17.2., 25.31., 26.31., 27.5., 28.31., 29.26., 30.31.
32	1.4., 5.32., 6.32., 7.32.	10.2., 11.32., 12.32., 13.12	16.5., 23.32., 24.6.	17.3., 25.32., 26.32., 27.6., 28.32., 29.1., 30.32.
33	3.7., 5.33., 6.33., 7.33.	10.3., 11.33., 12.33., 13.13.	16.6., 23.33., 24.7.	17.4., 25.3., 26.33., 27.7., 28.33., 29.2., 30.33.
34	3.8., 5.34., 6.34., 7.34.	10.4., 11.34., 12.34., 13.14.	16.7., 23.34., 24.8.	17.5., 25.34., 26.34., 27.8., 28.34., 29.3., 30.34.
35	3.9., 5.35., 6.10., 7.35.	8.2., 11.35., 12.35., 13.15.	15.4., 23.35., 24.9.	17.6., 25.35., 26.35., 27.9., 28.35., 29.4., 30.35.

Variant	Deadlines for the CIPIP (1Ind semester)	
	Until the implementation of module1	**Until the implementation of module2**
1	1.27; 2.18; 3.23; 4.1; 5.26; 6.1	9.24; 10.17; 11.22; 12.1; 13.23
2	1.28; 2.19; 3.24; 4.2; 5.27; 6.2	9.25; 10.18; 11.23; 12.2; 13.24
3	1.29; 2.20; 3.25; 4.3; 5.28; 6.3	9.26; 10.19; 11.24; 12.3; 13.25
4	1.30; 2.21; 3.26; 4.4; 5.29; 6.4	9.27; 10.20; 11.25; 12.4; 13.26
5	1.31; 2.22; 3.27; 4.5; 5.30; 6.5	9.28; 10.21; 11.26; 12.5; 13.27
6	1.32; 2.23; 3.28; 4.6; 5.31; 6.6	9.29; 10.22; 11.27; 12.6а; 13.28
7	1.33; 2.24; 3.29; 4.7; 5.32; 6.7	9.30; 10.23; 11.28; 12.6б; 13.29
8	1.34; 2.25; 3.30; 4.8; 5.33; 6.8	9.31; 10.24; 11.29; 12.6в; 13.30
9	1.35; 2.26; 3.31; 4.9; 5.34; 6.9	9.32, 10.25; 11.30; 12.7; 13.31
10	1.1; 2.27; 3.32; 4.10; 5.35; 6.10	9.33; 10.26; 11.31; 12.8а; 13.32
11	1.2; 2.28; 3.33; 4.11; 5.1; 6.11	9.34; 10.27; 11.32; 12.8б; 13.33
12	1.3; 2.29; 3.34; 4.4; 5.2; 6.12	9.35; 10.28; 11.33; 12.9; 13.34
13	1.4; 2.30; 3.35; 4.2; 5.3; 7.1	9.1; 10.29; 11.34; 12.10; 13.35
14	1.5; 2.31; 3.1; 4.3; 5.4; 7.2	9.2; 10.30; 11.35; 12.11; 13.1
15	1.6; 2.32; 3.2; 4.4; 5.5; 7.3	9.3; 10.31; 11.1; 12.12а; 13.2
16	1.7; 2.33; 3.3; 4.5; 5.6; 7.4	9.4; 10.32; 11.2; 12.12б; 13.3
17	1.8; 2.34; 3.4; 4.6; 5.7; 7.5	9.5; 10.33; 11.3; 12.12в; 13.4
18	1.9; 2.35; 3.5; 4.7; 5.8; 7.6	9.6; 10.34; 11.4; 12.12г, 13.5
19	1.10; 2.1; 3.6; 4.8; 5.9; 7.7	9.7; 10.35; 11.5; 12.13а; 13.6
20	1.11; 2.2; 3.7; 4.9; 5.10; 7.8	9.8; 10.1; 11.6; 12.13б; 13.7
21	1.12; 2.3; 3.8; 4.10; 5.11; 7.9	9.9; 10.2; 11.7; 12.14а; 13.8
22	1.13; 2.4; 3.9; 4.11; 5.12; 7.10	9.10; 10.3; 11.8; 12.14б; 13.9
23	1.14; 2.5; 3.10; 4.1; 5.13; 7.11	9.11; 10.4; 11.9; 12.15; 13.10
24	1.15; 2.6; 3.11; 4.2; 5.14; 7.12	9.12; 10.5; 11.10; 12.1; 13.11
25	1.16; 2.7; 3.12; 4.3; 5.15; 8.1	9.13; 10.6; 11.11; 12.2; 13.12
26	1.17; 2.8; 3.13; 4.4; 5.16; 8.2	9.14; 10.7; 11.12; 12.3; 13.18
27	1.18; 2.9; 3.14; 4.4; 5.17; 8.3	9.15; 10.8; 11.13; 12.4; 13.14
28	1.19; 2.10; 3.15; 4.5; 5.18; 8.4	9.16; 10.9; 11.14; 12.5; 13.15
29	1.20; 2.11; 3.16; 4.6; 5.19; 8.5	9.17; 10.10; 11.15; 12.6а; 13.16
30	1.21; 2.12; 3.17; 4.7; 5.20; 8.6	9.18; 10.11; 11.16; 12.7; 13.17
31	1.22; 2.13; 3.18; 4.8; 5.21; 8.7	9.19; 10.12; 11.17; 12.8а; 13.18
32	1.23; 2.14; 3.19; 4.9; 5.22; 8.8	9.20; 10.13; 11.18; 12.9; 13.19
33	1.24; 2.15; 3.20; 4.10; 5.23; 8.9	9.21; 10.14; 11.19; 12.10; 13.20
34	1.25; 2.16; 3.21; 4.11; 5.24; 8.10	9.22; 10.15; 11.20; 12.11; 13.21
35	1.26; 2.17; 3.22; 4.12; 5.25; 8.11	9.23; 10.16; 11.21; 12.12б; 13.22

Criteria for evaluating a complex practical, individual task

Complex practical individual task is evaluated according to the stoical scale and makes up 15-20% of the final score in the discipline "Higher Mathematics".

• "excellent" (90-100 points) is exhibited if the student has fully completed the KPIZ (he answered the theoretical questions and solved all problems, can justify their solution).

• "good" (75-89 points) is exhibited if the student fully performed the CIPIZ, but when he covered the theoretical issues or when solving certain tasks, he made mistakes.

• "satisfactorily" (60-74 points) is exhibited if the student fulfilled the CIPIZ, but can not, without the help of others, make appropriate substantiation of theoretical and practical tasks, can not draw the correct conclusions when solving economic problems.

• "unsatisfactory" (less than 60 points) is exhibited if the student fulfills the written version of the KPIZ at a satisfactory level, but does not know the answers to theoretical questions, can not explain the solution of his practical tasks, can not draw any conclusions in solving problems. economic problems.

In case of unsatisfactory assessment, the student completes the KPIZ and is appropriately prepared for re-defense.

List of recommended sources

1. Apanasov P.T., Apanasov N.P. Collection of mathematical problems with practical content: Kn. for the teacher. - Moscow: Enlightenment, 1987. - 110 p.

2. Barkovskii V.V., Barkovskaya N.V. Mathematics for economists: Higher mathematics. □ K .: National Academy of Management, 1997. - 397 pp.

3. Belinsky VA, Kalichman I. L., Maystrov L. E., Mit'kin A. M. Higher mathematics with the basics of mathematical statistics. - M .: Vyssh. Shk., 1965. - 516 pp.

4. Bugir M.K. Mathematics for economists: Textbook. □ Ternopil: Textbooks and manuals, 1998. - 192 p.

5. Valiev K. G., Galladova I. A. Higher mathematics: Textbook: In 2 ch. □ K .: KNEU, 2001. □ Part 1. □ 546 pp .; - K .: KNEU, 2002.-Ch.2.-451c.

6. Higher Mathematics: Teaching Method. Manual for independent study of discipline / K. G. Valiev, I. A. Galladova, O. I. Lyuty and others. □ K .: KNEU, 1999. □ 396 p.

7. Higher mathematics. Textbook / Dombrovsky V.A., Kryzhanivsky I.M., Matskiv R. S. and others; edited by Shinkarik M.I. - Ternopil: Karpyuk's View, 2003. - 480 p.

8. Glagolev AA, Solntseva T. Course of higher mathematics. - Izd. 2nd, pererab. and add - M .: Vyssh. Shk., 1971. □ 656 p.

9. Gudimenko F. S., Borisenko D. M., Volkova V. O. and others. Collection of Problems in Higher Mathematics: A Manual. □ K .: View of the KSU, 1967. □ 352 p.

10. Danko P. E., Popov AG Higher mathematics in exercises and tasks: Textbook. allowance □ Izd. 2nd □ M .: Exit Shk., 1974.- Ch. I. - 416 p.

11. Danko P. E., Popov AG Higher mathematics in exercises and tasks: Textbook. allowance □ Izd. 2nd □ M .: Exit Shk., 1974. - Ch. II.- 464 c.

12. Dobrozhitskaya I. G., Dobrozhitsky M. B. A brief guide to solving problems in higher mathematics (for technical schools) .- Minsk: Vyshesh. Shk., 1972. - 200 p.

13. Kaplan IA Practical classes in higher mathematics: Textbook. allowance □ Izd. 4th Kharkiv: Izvst. Of KhSU, 1970. - Ch. I, II.- 576 p.

14. Kaplan IA Practical classes in higher mathematics: Textbook. allowance □ Izd. 3rd, stereotyped. Part III. □ Izd. 2nd, stereotyped. □ Ch. IV. □ Kharkiv: Publishing house, KhGU, 1971. □ 500 p.

15. Karasev AI, Aksyutina Z. M., Savelyeva T. I. The course of higher mathematics for the economic universities. □ Ch. I. Fundamentals of Higher Mathematics: Study. allowance □ M .: Exit Shk., 1982.- 272 pp.

16. Klyueva L. A., Talsky, D. A. Practice on mathematics for correspondence technical schools: Textbook. allowance □ M .: Exit Shk., 1970. 446 pp.

17. Krynsky H. E. Mathematics for economists: Per. from polsk Menikera V. D., ed. Barengoltsa M.I. - Moscow: Statistics, 1970. - 584 p.

18. Minorsky VP The collection of problems in higher mathematics: Textbook. allowance □ M .: Nauka, 1971. □ 352 p.

19. Nemish VM, Protsik A.I., Berezkaya K.M. Workshop on Higher Mathematics: Teaching. manual., 3rd edition. - Ternopil: TNEU in "Economic Thought" 2010. □ 304 p.

20. Petrov VA Mathematical problems from agricultural practice: A manual for teachers. - Moscow: Enlightenment, 1980. 64 p.

21. Collection of tasks in the course of higher mathematics: Textbook. allowance for woven garments. / Kruckovich G.I., Gutarina N.I., Dubyuk P. E., et al., Ed. Kryuchkovich G.I. □ Izd. 3rd, pererab. □ M .: Exit Shk., 1973. □ 576 pp.

22. A reference manual for mathematical analysis. Ch.I. Introduction to the analysis, the derivative, the integral / Lyashko I. I., Boyarchuk A. K., Gai Y. G. - K .: Vysshaya Shk., 1978.- 696 p.

23. Typical Individual Settlement Problems in Higher Mathematics: Textbook / Dombrovsky IV, Lesik O.F., Migovich F.M. etc.; edited by Shinkarika MI - Ternopil: in "Tutorials and manuals", 2008.-208s.

24. Fisher S., Dornbusch R., Shmalenzi R. Economics: Per. from english from 2nd ed. - Moscow: "Case of LTD.", 1993. - 864 p.

Variant **№1.1**

1. Calculate the determinant: $\begin{vmatrix} 1 & 2 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 5 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} 2x_1 + 3x_2 + 2x_3 = 1 \\ 4x_1 + x_2 - 3x_3 = 0 \\ x_1 - 3x_2 + x_3 = 5 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	7	1	3	2	2
S_2	7	2	1	2	3
S_3	7	2	2	1	2

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. *A(-3;1), B(7;2), C(4;4).*

6. Build a circle $2x^2+2y^2-8x+5y-4=0$.

7. Find the limits of functions: а) $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4}$; б) $\lim_{x\to 10}\frac{\sqrt{x-1}-3}{x-10}$; в) $\lim_{n\to\infty}\frac{3n}{1-2n}$; г) $\lim_{x\to 0}\frac{tg\ 3x}{sin\ 2x}$;

д) $\lim_{x\to\infty}\left(\frac{1+x}{2+x}\right)^x$.

8. Find derivative functions: а) $y = x\arccos\frac{x}{2} - \sqrt{4-x^2}$; б) $y = \ln\sqrt{\frac{1+\sin x}{1-\sin x}}$; в) $y = \left(\frac{1-2x}{7+3x}\right)^2$.

9. The enterprise produces *x* units of products per month. Total production costs are described by function $f_1(x) = \frac{1}{15}x^2 + 53x + 400$. $f_2(x) = 22 - \frac{1}{16}x$ - the relationship between the specific price and the number of units of product *x* that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y = \frac{x}{x^2-4}$.

Variant **№1.2 .**

1. Calculate the determinant: $\begin{vmatrix} 1 & 0 & 3 & 4 \\ 2 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 4 & 2 & -1 & 0 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 + 3x_2 - 2x_3 = 2 \\ x_1 + x_2 + x_3 = 3 \\ 2x_1 - x_2 + x_3 = 2 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	9	2	3	2	3
S_2	10	1	4	3	3
S_3	10	3	3	2	4

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4.The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. *A(-1;1) , B(3;2) , C(3;-2)*

6. Make an canonical ellipse equation that passes through points M(*-5;-4)* and $N\left(5\sqrt{0,5};\ 2\sqrt{6}\right)$.

7. Find the limits of functions: а) $\lim\limits_{x\to 1}\frac{x^3-6x^2+11x-6}{x^2-3x+2}$;б) $\lim\limits_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$;в) $\lim\limits_{n\to\infty}\frac{5n+2}{2+n}$; г) $\lim\limits_{x\to 0}\frac{x ctg\ 4x}{cos\ 5x}$;

д) $\lim\limits_{x\to\infty}\left(\frac{x+3}{3x-1}\right)^x$.

8. Find derivative functions:а) $y=\frac{1}{2\sqrt{x}}arcsin3x$; б) $y=ln\left(\frac{2x-3}{x^2+1}\right)$; в) $y=\left(\frac{1}{4}x^3+7x^2\right)^3$.

9. The enterprise produces *x* units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{40}x^2+8x+300$. $f_2(x)=30-\frac{1}{10}x$ - the relationship between the specific price and the number of units of product *x* that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y=\frac{x^2}{x-2}$.

Variant №**1.3.**

1. Calculate the determinant: $\begin{vmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 1 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 + x_2 - 3x_3 = -3 \\ 2x_1 - 2x_2 - x_3 = 5 \\ 3x_1 - x_2 + x_3 = 3 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	11	1	4	3	3
S_2	12	2	3	2	4
S_3	7	4	1	1	2

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(1;-2) , B(5;2) , C(6;-1) .

6. The eccentricity of hyperbole equals $\frac{\sqrt{17}}{3}$**.** Make an equation of hyperbola that passes through a point $A(3\sqrt{2};2\sqrt{2})$**.**

7. Find the limits of functions: a) $\lim\limits_{x\to\infty}\frac{3x^4+x^2-5x+6}{x^3+3x^2+7x-1}$;б) $\lim\limits_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}$;

в) $\lim\limits_{x\to 0}\left(\frac{2+x}{5-x}\right)^x$; г) $\lim\limits_{x\to 6}\frac{2-\sqrt{x-2}}{x^2-36}$; д) $\lim\limits_{x\to 0}\frac{\cos 3x}{x\,ctg\ 2x}$.

8. Find derivative functions: a) $y=\sin 2x\sqrt{4-3x^2}$; б) $y=\ln\sqrt{\frac{x^3-1}{3x+1}}$; в) $y=x\,tg\,5x+3^{x-3}$;

9. The enterprise produces *x* units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{40}x^2+25x+200$. $f_2(x)=30-\frac{1}{10}x$ - the relationship between the specific price and the number of units of product *x* that can be sold at that price. Calculate the conditions under which the profit will be maximized.

10. Explore the function and build its graph $y=\frac{x^2-6x+13}{x-3}$.

Variant №1. 4.

1. Calculate the determinant: $\begin{vmatrix} 0 & 1 & 2 & 4 \\ 2 & 1 & 3 & -1 \\ 3 & 1 & 0 & 2 \\ -1 & 1 & 4 & 1 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 - x_2 + x_3 = 3 \\ -x_1 + 2x_2 - 3x_3 = -7 \\ 3x_1 + 5x_2 + x_3 = 5 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	10	1	3	2	2
S_2	6	2	1	1	3
S_3	10	3	1	2	4

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5.A(-4;3) , B(2;7) , C(1;2) .

6. Find the equation of a circle symmetric with a circle $x^2 + y^2 = 2x + 4y - 4$ relative to the straight line $x - y - 3 = 0$.

7. Find the limits of functions: a) $\lim\limits_{n\to\infty} \frac{3n}{1-2n}$; б) $\lim\limits_{x\to -2} \frac{x^2-3x-10}{x^2-4}$; в) $\lim\limits_{x\to -8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}$; г) $\lim\limits_{x\to 0} \frac{tg\ 5x}{3x}$;

д) $\lim\limits_{x\to\infty} \left(\frac{x}{x+1}\right)^{3x}$.

8. Find derivative functions: a) $y = \frac{x}{2} \arcsin \frac{x}{5}$; б) $y = \frac{1}{3} \ln \frac{tgx+1}{tgx-1}$; в) $y = \left(\frac{x}{1-5x}\right)^2$.

9. The enterprise produces *x* units of products per month. Total production costs are described by function функцією $f_1(x) = \frac{1}{48}x^2 + 24x + 100$. $f_2(x) = 38 - \frac{1}{8}x$- the relationship between the specific price and the number of units of product *x* that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y = \frac{x^3}{3} - x^2 - 3x + 7$.

Variant №1.5 .

1. Calculate the determinant: $\begin{vmatrix} 1 & 1 & 2 & 2 \\ 1 & 4 & -1 & -1 \\ 3 & 1 & 3 & 3 \\ 6 & 2 & 0 & 2 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 - 3x_2 + 4x_3 = 3 \\ 2x_1 - 3x_2 - x_3 = 0 \\ 3x_1 - x_2 - x_3 = 4 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	7	1	2	1.5	3
S_2	10	2	4	1	2
S_3	11	2	2	3	3

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(-2;1),B(3;5),C(2;-1).

6. Make a circle equation that passes through a point A(5;0),B(1;4), if the center lies on the straight line $x+y-3=0$.

7. Find the limits of functions:а) $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$; б) $\lim\limits_{x\to 0}\frac{tgx}{3x}$; в) $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-5}\right)^{2x}$;

г) $\lim\limits_{x\to 1}\frac{x^3-3x^2+2x}{2x^2-2}$; д) $\lim\limits_{n\to\infty}\frac{5n+2}{2+n}$.

8. Find derivative functions: а) $y=\frac{\sin\sqrt{5x}}{2\sqrt{x}}$; б) $y=\ln\left(\frac{5x}{e^{2x}-e^{-2x}}\right)$; в) $y=\sqrt[3]{tg^2 3x}$.

9. The enterprise produces x units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{24}x^2+12x+200$. $f_2(x)=38-\frac{1}{15}x$ - the relationship between the specific price and the number of units of product x that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y=\frac{5}{x^2-9}$.

Variant №**1. 6.**

1. Calculate the determinant: $\begin{vmatrix} 0 & 1 & 1 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 + x_2 - x_3 = 0 \\ 2x_1 - x_2 + x_3 = 3 \\ -x_1 + 5x_2 - 2x_3 = -3 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	8	1	3	2	2
S_2	11	2	1	4	3
S_3	9	3	3	2	1

Визначити кількість одиниць продукції P_1, P_2, P_3, P_4 якщо ресурси повністю вичерпані.Вказати базовий розв'язок..

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(1;1) , B(5;5) , C(7;1) .

6. Make a straight line equation that passes through the hyperbolic tricks $7x^2 - 5y^2 = 35$ and form with the axis OX an angle 60^0 . Make a drawing.

7. Find the limits of functions: а) $\lim\limits_{x\to 2}\frac{x^2-7x+10}{x^2-8x+12}$; б) $\lim\limits_{n\to\infty}\frac{3n^2-3n+2}{3+n^2}$; в) $\lim\limits_{x\to 3}\frac{2-\sqrt{1+x}}{x^2-9}$ г) $\lim\limits_{x\to 0}\frac{3x}{\sin 5x}$;

д) $\lim\limits_{x\to\infty}\left(\frac{n-1}{6n+1}\right)^{2n}$.

8. Find derivative functions:а) $y = \arcsin\sqrt{1-x^2}$;б) $y = \ln^3\left(1+e^{x/3}\right)$;в) $y = x tg 5x + 3^{x-3}$.

9. The enterprise produces x units of products per month. Total production costs are described by function $f_1(x) = \frac{1}{50}x^2 + 15x + 400$. $f_2(x) = 51 - \frac{1}{10}x$ - the relationship between the specific price and the number of units of product x that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y = x + e^{-x}$.

Variant №1.7

1. Calculate the determinant: $\begin{vmatrix} 2 & 3 & 1 & 1 \\ 3 & -1 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 4 & 0 & 1 & 4 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} 2x_1 - x_2 + x_3 = 3 \\ x_1 + x_2 + 3x_3 = 12 \\ -2x_1 - x_2 + 2x_3 = 2 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	11	2	2	3	1
S_2	7	1	4	1	2
S_3	7	3	2	2	3

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(-4;2) , B(0;2) , C(7; 0) .

6. Find the eccentricity of an ellipse if its axis is as 13:5 .

7. Find the limits of functions:a) $\lim\limits_{x\to -3}\frac{x^2+3x}{9-x^2}$; б) $\lim\limits_{x\to\infty}\frac{4x^2-3}{5x^2-6}$;

в) $\lim\limits_{x\to 0}\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{x^2-x}$; г) $\lim\limits_{x\to 0}\frac{\sin^2\frac{x}{3}}{x^2}$; д) $\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+1}\right)^{-3n}$.

8. Find derivative functions:a) $y=\sqrt{1+\sin 4x}$;б) $y=\ln\sqrt{\frac{x^3-7}{x^3+1}}$;в) $y=e^{\sin x-\cos x}\cdot\sin x$.

9. The enterprise produces x units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{44}+14x+200$. $f_2(x)=42-\frac{1}{12}x$ - the relationship between the specific price and the number of units of product x that can be sold at that price. Calculate the conditions under which the profit will be maximized.

10. Explore the function and build its graph $y=\frac{1}{2x-x^2}$.

Variant №1.8

1. Calculate the determinant: $\begin{vmatrix} 1 & 0 & 2 & 1 \\ -1 & 2 & -1 & 1 \\ 4 & 1 & 1 & 0 \\ 3 & -3 & 4 & 2 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 + x_2 + x_3 = 3 \\ 3x_1 - x_2 - 2x_3 = -5 \\ 2x_1 + x_2 - x_3 = -1 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	7	1	3	2	2
S_2	9	2	1	4	3
S_3	8	2	3	2	4

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(3;-3) , B(5;2) , C(7; 0) .

6. Through the focus points of the parabola $y^2=8x$ and through its point, the abscissa of which is equal to 0.5; and the ordinate is positive, a straight line is made. Find the distance from the center of the circle $x^2+y^2+6x+4y-3=0$ to this line.

7. Find the limits of functions: а) $\lim_{x\to -1} \frac{x^2-3x-4}{x^2+3x+2}$; б) $\lim_{x\to 2} \frac{\sqrt{1+x+x^2}-\sqrt{7+2x-x^2}}{x^2-2x}$;

в) $\lim_{x\to\infty} \frac{3-7x^3}{4+x^4}$; г) $\lim_{x\to 0} \frac{\sin 2x}{\sqrt{x+1}-1}$; д) $\lim_{x\to\infty} \left(\frac{n+3}{n+1}\right)^{2n}$.

8. Find derivative functions: а) $y=(9x^2+1)\operatorname{arctg} 3x$;б) $y=\ln\frac{1+\cos^3 x}{1+\sin 3x}$;в) $y=\cos\sqrt[3]{2x+1}$.

9. The enterprise produces *x* units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{54}x^2+15x+800$. $f_2(x)=47-\frac{1}{10}x$ - the relationship between the specific price and the number of units of product *x* that can be sold at that price. Calculate the conditions under which the profit will be maximized. Determine marginal and total costs, profit under these conditions.

10. Explore the function and build its graph $y=\frac{3x^4}{4}-x^3-9x^2+7$.

Variant №1.9

1. Calculate the determinant: $\begin{vmatrix} 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & 0 \\ 1 & -3 & 4 & 2 \\ -4 & 0 & 7 & 1 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} 4x_1 + x_2 + x_3 = 6 \\ x_1 - 2x_2 - x_3 = -1 \\ 2x_1 + x_2 - 3x_3 = -4 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	7	1	3	2	4
S_2	9	2	1	4	2
S_3	9	3	3	2	1

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. A(5;2) , B(9;3) , C(8;-2) .

6. The ellipse passes through the point M (1; 1) and has an eccentricity. Put the ellipse equation.

7. Find the limits of functions: а) $\lim\limits_{n\to\infty}\frac{3n^4+5n^2+2}{1+n^3}$; б) $\lim\limits_{x\to-1}\frac{\sqrt{3-x}-2}{1+\sqrt[3]{x}}$; в) $\lim\limits_{x\to0}\frac{1-\cos 2x}{x\sin x}$; г) $\lim\limits_{x\to0}\frac{arctgx}{x}$; д) $\lim\limits_{n\to\infty}\left(\frac{3x+4}{3x-7}\right)^{2x-5}$.

8. Find derivative functions:а) $y=(1+\ln\cos x)^3$; б) $y=\ln\sqrt{\frac{1+\sin x}{1-\sin x}}$; в) $y=\frac{\cos x}{x^2}$.

9. The enterprise produces *x* units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{30}x^2+8x+300$. $f_2(x)=40-\frac{1}{10}x$ - залежність між питомою ціною і кількістю одиниць продукції *x*, яку можна продати по цій ціні. Розрахувати, за яких умов прибуток буде максимальним. Визначити маргінальні і сумарні витрати, прибуток при цих умовах.

10. Explore the function and build its graph $y=\frac{x}{4}+\frac{4}{x}$.

Variant №1.10

1. Calculate the determinant: $\begin{vmatrix} -1 & 2 & 4 & 0 \\ -1 & 1 & 2 & 1 \\ 2 & -1 & 3 & 4 \\ 1 & 2 & 0 & 2 \end{vmatrix}$.

2. Solve the system of equations (by three methods): $\begin{cases} x_1 - x_2 - x_3 = 2 \\ 2x_1 + x_2 - 3x_3 = 1 \\ 3x_1 + 2x_2 + x_3 = 1 \end{cases}$.

3. Three types of raw materials S1, S2, and S3 are used for the production of four types of products P1, P2, P3, P4. Inventories and cost per unit of production are shown in the table:

Type of raw material	Stocks of raw materials	Consumption of raw materials per unit of production			
		P_1	P_2	P_3	P_4
S_1	9	2	1	2	3
S_2	12	3	2	2	4
S_3	12	1	3	2	2

Determine the number of units of products P1, P2, P3, and P4 if the resources are completely exhausted. Provide a basic solution .

4. The three firms produced four types of products A1, A2, A3, and A4. Accordingly I - 8 pcs .; 0 pcs .; 5 pcs .; 14 pcs .; II -5; 4; 3; 50; III - 2; 0; 2; 80. Price 1 pc. products in city B1, respectively - 7 UAH, 3 UAH, 2 UAH, 2.1 UAH, in B2 - 3; 0.4; 2.3; 1.3, in B3 - 3; 7; 3.5; 7. Determine the revenue that firms will receive from selling this product in each of the cities. (Use matrix product).

5. **A(2;2), B(1;6), C(2;3)**

6. Make a circle equation that passes through points A(3;1), B(5;3), if the center lies on the line $x=y$.

7. Find the limits of functions:а) $\lim\limits_{x\to -3}\frac{2x^2+4x-6}{x^2-9}$; б) $\lim\limits_{x\to 0}\frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$;

в) $\lim\limits_{x\to\infty}\frac{3x^5-2x^3+5x+2}{2x-5x^2}$;г) $\lim\limits_{x\to 0}\frac{\sin\frac{x}{5}}{x}$; д) $\lim\limits_{x\to\infty}\left(\frac{5x}{1+5x}\right)^x$.

8. Find derivative functions:

а) $y=3^{\sin x}-\sqrt[3]{x^2+1}$;б) $y=\ln(x^3-2x^2+1)$; в) $y=\left(1-\frac{x^2}{3}\right)^5$.

9. The enterprise produces x units of products per month. Total production costs are described by function $f_1(x)=\frac{1}{48}x^2+10x+200$. $f_2(x)=39-\frac{1}{10}x$ - the relationship between the specific price and the number of units of product x that can be sold at that price. Calculate the conditions under which the profit will be maximized.

10. Explore the function and build its graph $y=\frac{7}{x^2-5x+4}$.

A sample of replies to the tasks of CPIT

Task 1. To calculate determinant of the second orde:

$$\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix}.$$

Untiing.

$$\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = 3 \times 1 - (-4) \times 2 = 3 + 8 = 11.$$

Task 2. To calculate determinant of the third order

:

$$\begin{vmatrix} 2 & 1 & -3 \\ 3 & 0 & -1 \\ 4 & -2 & 5 \end{vmatrix}.$$

Untiing.

$$\begin{vmatrix} 2 & 1 & -3 \\ 3 & 0 & -1 \\ 4 & -2 & 5 \end{vmatrix} = 2 \times 0 \times 5 + 1 \times (-1) \times 4 + (-3) \times 3 \times (-2) - (-3) \times 0 \times 4 - \\ -1 \times 3 \times 5 - 2 \times (-1) \times (-2) = 0 - 4 + 18 + 0 - 15 - 4 = \\ = -5.$$

Task 3. To calculate determinant of the third order, decomposing him after the elements of line (or column):

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 0 \\ 1 & 2 & -5 \end{vmatrix}.$$

Untiing.

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 0 \\ 1 & 2 & -5 \end{vmatrix} = 3 \times (-1)^{2+1} \times \begin{vmatrix} 2 & 4 \\ 2 & -5 \end{vmatrix} + (-1) \times (-1)^{2+2} \times \begin{vmatrix} 1 & 4 \\ 1 & -5 \end{vmatrix} +$$

$$+ 0 \times (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 3 \times (-1)^3 \times (-10 - 8) - 1 \times (-1)^4 \times (-5 - 4) + 0 =$$

$$= -3 \times (-18) - 1 \times (-9) = 63.$$

Task 4.To calculate determinant of fourth order, using him to property:

$$\begin{vmatrix} 3 & 2 & 1 & 4 \\ 1 & 0 & 1 & 2 \\ -1 & -1 & 3 & 0 \\ 0 & 2 & 1 & 5 \end{vmatrix}.$$

Untiing.

$$\begin{vmatrix}3&2&1&4\\1&0&1&2\\-1&-1&3&0\\0&2&1&5\end{vmatrix}=\begin{vmatrix}0&2&-2&-2\\1&0&1&2\\0&-1&4&2\\0&2&1&-5\end{vmatrix}=(-1)^3\begin{vmatrix}2&-2&-2\\-1&4&2\\2&1&-5\end{vmatrix}=$$

$$=-\begin{vmatrix}2&0&0\\-1&3&1\\2&3&-3\end{vmatrix}=-2\times(-1)^2\times\begin{vmatrix}3&1\\3&-3\end{vmatrix}=-2\times(-9-3)=24$$

.

Task 4. Find a product $A\cdot B$ if

$$A=\begin{bmatrix}1&2&3\\4&5&6\\2&1&4\end{bmatrix},\ B=\begin{bmatrix}-2&1&2\\3&2&1\\1&3&2\end{bmatrix}.$$

Solution.

$$A\cdot B=\begin{bmatrix}1&2&3\\4&5&6\\2&1&4\end{bmatrix}\cdot\begin{bmatrix}-2&1&2\\3&2&1\\1&3&2\end{bmatrix}=$$

$$=\begin{bmatrix}1\cdot(-2)+2\cdot3+3\cdot1&1\cdot1+2\cdot2+3\cdot3&1\cdot2+2\cdot1+3\cdot2\\4\cdot(-2)+5\cdot3+6\cdot1&4\cdot1+5\cdot2+6\cdot3&4\cdot2+5\cdot1+6\cdot2\\2\cdot(-2)+1\cdot3+4\cdot1&2\cdot1+1\cdot2+4\cdot3&2\cdot2+1\cdot1+4\cdot2\end{bmatrix}=$$

$$=\begin{bmatrix}7&14&10\\13&32&25\\3&16&13\end{bmatrix}.$$

Task **5.** Find a matrix rank

$$A=\begin{bmatrix}1&2&-1&3\\3&6&-3&-1\\3&6&-3&10\end{bmatrix}.$$

Solving The rank of the matrix will be searched by the elementary transformation method.

$$A=\begin{bmatrix}1&2&-1&3\\3&6&-3&-1\\3&6&-3&10\end{bmatrix}\ (-3)\ \Leftrightarrow\begin{bmatrix}1&2&-1&3\\0&0&0&-10\\0&0&0&1\end{bmatrix}\Leftrightarrow$$

$$\Leftrightarrow\begin{bmatrix}1&2&-1&3\\0&0&0&1\\0&0&0&-10\end{bmatrix}\ 10\ \Leftrightarrow\begin{bmatrix}1&2&-1&3\\0&0&0&1\\0&0&0&0\end{bmatrix}\Leftrightarrow$$

$$\Leftrightarrow\begin{bmatrix}1&3&-1&2\\0&1&0&0\\0&0&0&0\end{bmatrix}.$$

Hence it follows that the rank of this matrix is 2 (below the main diagonal □ zeros and two elements of the main diagonal), $\mathrm{rang}(A)=2$

Task **6.** Find an inverse matrix to a matrix

$$A = \begin{bmatrix} -2 & 3 & 4 \\ 3 & -1 & -3 \\ -1 & 2 & 2 \end{bmatrix}.$$

Solving. First, make sure the matrix has a reverse A^{-1}. Identifier

$$|A| = \begin{vmatrix} -2 & 3 & 4 \\ 3 & -1 & -3 \\ -1 & 2 & 2 \end{vmatrix} = 4 + 24 + 9 - 4 - 18 - 12 = 3 \neq 0.$$

So, the matrix has an inverse. We find algebraic additions to the elements of the matrix:

$$A_{11} = (-1)^2 \begin{vmatrix} -1 & -3 \\ 2 & 2 \end{vmatrix} = -2 - (-6) = 4;$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & -3 \\ -1 & 2 \end{vmatrix} = -(6 - 3) = -3;$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = 6 - 1 = 5;$$

$$A_{21} = -\begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} = 2; \qquad A_{22} = \begin{vmatrix} -2 & 4 \\ -1 & 2 \end{vmatrix} = 0;$$

$$A_{23} = -\begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} = 1; \qquad A_{31} = \begin{vmatrix} 3 & 4 \\ -1 & -3 \end{vmatrix} = -5;$$

$$A_{32} = -\begin{vmatrix} -2 & 4 \\ 3 & -3 \end{vmatrix} = 6; \qquad A_{33} = \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = -7.$$

The matrix of algebraic additions will be

$$\overline{A} = \begin{bmatrix} 4 & -3 & 5 \\ 2 & 0 & 1 \\ -5 & 6 & -7 \end{bmatrix}.$$

The attached matrix has the form:

$$A^* = \begin{bmatrix} 4 & 2 & -5 \\ -3 & 0 & 6 \\ 5 & 1 & -7 \end{bmatrix}.$$

So, we get

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 2 & -5 \\ -3 & 0 & 6 \\ 5 & 1 & -7 \end{bmatrix}.$$

Task 7. To solve the system of equations according to the Cramer's rule

$$\begin{cases} x_1 + 2x_2 - x_3 = -3 \\ 2x_1 + 3x_2 + x_3 = -1 \\ x_1 - x_2 - x_3 = 3 \end{cases}.$$

Розв'язування.

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} = -3 + 2 + 2 + 3 + 4 + 1 = 9 \neq 0.$$

$$x_1 = \frac{\Delta_1}{\Delta},\ x_2 = \frac{\Delta_2}{\Delta},\ x_3 = \frac{\Delta_3}{\Delta}.$$

$$\Delta_1 = \begin{vmatrix} -3 & 2 & -1 \\ -1 & 3 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 9 - 1 + 6 + 9 - 2 - 3 = 18,$$

$$\Delta_2 = \begin{vmatrix} 1 & -3 & -1 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = 1 - 6 - 3 - 1 - 6 - 3 = -18,$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 9 + 6 - 2 + 9 - 12 - 1 = 9.$$

$$x_1 = \frac{18}{9} = 2;\ \ x_2 = \frac{-18}{9} = -2;\ \ x_3 = \frac{9}{9} = 1.$$

So, the solution of the given system will be (2; –2; 1).

Task **8.** Solve a matrix system with a system of equations

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 = 9 \\ x_1 + 2x_2 - 3x_3 = 14 \\ 3x_1 + 4x_2 + x_3 = 16 \end{cases}.$$

Розв'язування.

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{bmatrix}.$$

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{vmatrix} = 4 + 8 - 27 - 12 - 3 + 24 = -6 \neq 0.$$

$$A_{11} = \begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix} = 14; \qquad A_{12} = -\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = -10;$$

$$A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2; \qquad A_{21} = 5; \qquad A_{22} = -4;$$

$$A_{23} = 1; \qquad A_{31} = -13; \qquad A_{32} = 8; \qquad A_{33} = 1.$$

$$\overline{A} = \begin{bmatrix} 14 & -10 & -2 \\ 5 & -4 & 1 \\ -13 & 8 & 1 \end{bmatrix}.$$

$$A^* = \begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix}.$$

$$A^{-1} = -\frac{1}{6}\begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix}.$$

$$X = -\frac{1}{6} \cdot \begin{bmatrix} 14 & 5 & -13 \\ -10 & -4 & 8 \\ -2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 14 \\ 16 \end{bmatrix} = -\frac{1}{6} \cdot \begin{bmatrix} 14 \cdot 9 + 5 \cdot 14 + (-13) \cdot 16 \\ (-10) \cdot 9 + (-4) \cdot 14 + 8 \cdot 16 \\ (-2) \cdot 9 + 1 \cdot 14 + 1 \cdot 16 \end{bmatrix} = = -\frac{1}{6} \cdot \begin{bmatrix} -12 \\ -18 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}.$$

The solution of the system will be: $x_1 = 2, \; x_2 = 3, \; x_3 = -2.$