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ARCH-BUILDING MODELS OF TIME SERIES PREDICTION FOR INVESTMENT
BUDOVA ARCH-MODELI SZEREGÓW CZASOWYCH DO PROGNOZOWANIA INWESTYCJI PORTFELOWYCH

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Summary: Nowadays, due to the increased internationalization of financial markets and the increasing openness of national economies, there is increasing cross-border movement of portfolio investments as a form of international economic relations. The objects of portfolio investment could be shares, bills, debt securities, derivatives etc. Direct and portfolio investments are different in nature and do not have the same influence on the economy of the resident. Since portfolio investments are providing the purchase of securities and other financial instruments and have a speculative nature due to the potential risk of rapid outflows, it can be concluded that they are largely volatile and, therefore, cannot be predicted by standard predictive models and ARIMA-models. The aim of our study is to attempt to apply econometric models of conditional and generalized heteroscedasticity (ARCH-model and GARCH-model) and to predict the volume of portfolio investments in Ukraine. The scope of portfolio investments in Ukraine is researched, and there is also built an econometric model of ARCH family. Evaluation of ARCH models options is made by maximum likelihood method. Checking the adequacy of models is done by selected the most optimal one and made for the forecasting period. Research was conducted using the package EViews 6.

Keywords: portfolio investments, conditional heteroskedasticity, volatility, maximum likelihood method, ARCH-models.

Streszczenie: Aktualnie wskutek internacjonalizacji rynków finansowych, jak również w wyniku coraz większej otwartości gospodarek narodowych dokonuje się aktywizacja przepływu transgranicznych inwestycji portfelowych jako jednej z postaci gospodarczych w wymiarze międzynarodowych. Przedmiotami inwestycji portfelowych mogą być akcje, weksle, dłużne papiery wartościowe, derywatywy itp. Inwestycje bezpośrednie oraz inwestycje portfelowe mają różne pochodzenie oraz w różnym stopniu oddziałują na gospodarkę kraju rezydenta. Ponieważ inwestycje portfelowe przewidują kupno papierów wartościowych oraz innych narzędzi finansowych i mają charakter spekulacyjny, wskutek potencjalnego ryzyka ich szybkiego zmniejszenia, można dojść do wniosku, że są one w znacznym stopniu wolatylne, a więc nie mogą być prognozowane za pomocą standardowych modeli prognozujących oraz ARIMA – modeli. Celem niniejszego badania jest próba stosowania modeli ekonometrycznych o heteroskedastyczności warunkowej oraz uogólnionej (ARCH-modele oraz GARCH-modele) dla prognozowania zasięgów inwestycji portfelowych na Ukrainę. Zostały zbadane wysokości inwestycji portfelowych na Ukrainie, opracowane modele ekonometryczne rodziny ARCH. Ocena parametrów modelu ARCH była prowadzona na podstawie metody maksymalnej wiarygodności. Modele zostały sprawdzone pod względem adekwatności, wybrano najbardziej optymalny z nich. Na podstawie tego modelu wykonano prognozowanie na jeden okres. Badanie przeprowadzono za pomocą pakietu EViews 6.

Słowa kluczowe: inwestycje portfelowe, heteroskedastyczność warunkowa, wolatylność, metoda maksymalnej wiarygodności, ARCH-modele.

1. Introduction

Nowadays, due to the increased internationalization of financial markets and the increasing openness of national economies, there is increasing cross-border movement of portfolio investments as a form of international economic relations.

The structure of foreign direct investments (hereinafter − FDI) includes: joint-stock (buying share of foreign company by direct foreign investor, at least 10% of the stated capital); intercompany loans and debt transactions; reinvested earnings; intangible assets such as technologies, trade marks (brands), managerial experience and more. The main reason for the implementation of FDI is to obtain a significant effect on the management of the business in which there is invested capital.

Portfolio investments represent capital that is invested by a resident of one country into shares and debt securities of companies in another country to obtain income. Objects of portfolio investment could be shares, bills, debt securities, derivatives etc. Main causes for the implementation of portfolio investments are profit and reduce the risk diversification of portfolio capital; they are financial transactions and are not accompanied by the transfer of intangible assets, management experience, know-how.

Direct and portfolio investments are different in nature and do not have the same influence on the economy of the resident. The main differences between them are:

- FDI, in contrast to a portfolio, give to investor the right to control.
- In most cases, direct investments are long-term investments, a portfolio, on the contrary – short-term.
- Various sources of funding and the motives of committing.
- Different effects on the economy and the state of the financial markets in the country of the investor and the recipient country.
- Portfolio investments are more destabilizing than FDI.
- Different levels of accessibility to the recipients.
- Various timing and degree of liquidity.

2. Problem statement and analysis of recent research

Taking into consideration the major differences between these types of investments, we consider it expedient to predict their streams separately from each other.

Since portfolio investments, as mentioned above, providing the purchase of securities and other financial instruments and have a speculative nature due to the potential risk of rapid outflows, it can be concluded that they are largely volatile and, therefore, cannot be predicted by standard predictive models and ARIMA-models.

The aim of our study is to attempt to apply econometric models of conditional and generalized heteroscedasticity (ARCH-model and GARCH-model) and to predict the volume of portfolio investments in Ukraine. The main difference between these models and standard approaches is that the predicted volatility (variance) is not constant, but depends on the previous states of the process, from the assessment level variance prior periods (GARCH-effect) and develops in time.

The tasks of estimation and forecasting of volatility are of considerable interest among economists. They were reviewed in their writing, in particular by such scholars as: T. Bollerslev,

R. Chou, K. Kroner [1992], C. Brownlees, R. Engle, B. Kelly [2012], R. Chou [1988], J. Ding, N. Meade [2010], F. Klaasen [Klaasen 2002].

The ARCH concept was introduced by Engle in 1982, realizing the idea that variance of the error in time depends on the square of errors in the previous periods. The ARCH process (*q*) is following [Engle 1982, p. 994]:

$$\sigma_t^2 = \mu + \gamma_1\varepsilon_{t-1}^2 + \gamma_2\varepsilon_{t-2}^2 + \ldots + \gamma_q\varepsilon_{t-q}^2 ,$$

For ARCH(1):

$$\sigma_t^2 = \mu + \gamma_1\varepsilon_{t-1}^2 .$$

It is assumed that, $\mu > 0$, $\gamma_k \geq 0$, the variance was not negative.

In 1986 Bollerslev proposed GARCH model. GARCH (*p*, *q*) is as follows [Bollerslev 1986, p. 310]:

$$\sigma_t^2 = \mu + \delta_1\sigma_{t-1}^2 + \delta_2\sigma_{t-2}^2 + \ldots + \delta_p\sigma_{t-p}^2 + \gamma_1\varepsilon_{t-1}^2 + \gamma_2\varepsilon_{t-2}^2 + \ldots + \gamma_q\varepsilon_{t-q}^2 =$$

$$= \mu + \sum_{k=1}^{p}\delta_k\sigma_{t-k}^2 + \sum_{k=1}^{q}\gamma_k\varepsilon_{t-k}^2 ,$$

$$\mu > 0,\ \gamma_k \geq 0,\ \delta_k \geq 0,$$

For GARCH(1,1):

$$\sigma_t^2 = \mu + \delta_1\sigma_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 .$$

In models GJR(*p*,*q*), the equation for the conditional variance is the following [Glosten, Jagannathan Runkle 199, p. 1783]:

$$\sigma_t^2 = \mu + \sum_{k=1}^{p}\delta_k\sigma_{t-k}^2 + \sum_{k=1}^{q}\gamma_k\varepsilon_{t-k}^2 + \sum_{k=1}^{q}L_k\, I(\varepsilon_{t-k} < 0)\varepsilon_{t-k}^2 ,$$

Where $I(\varepsilon_{t-k} < 0) = 1$, If $\varepsilon_{t-k} < 0$, i $I(\varepsilon_{t-k} < 0) = 0$ otherwise, constrained

$$\sum_{k=1}^{p}\delta_k + \sum_{k=1}^{q}\gamma_k + \frac{1}{2}\sum_{k=1}^{q}L_k < 1 ,$$

$$\mu \geq 0,\ \gamma_k \geq 0,\ \delta_k \geq 0,\ \gamma_k + L_k \geq 0 .$$

This model is often called TARCH(*p*,*q*) model (threshold ARCH).

In models EGARCH(*p*,*q*) the equation for the conditional variance is given [Nelson, p. 351]:

$$\ln\sigma_t^2 = \mu + \sum_{k=1}^{p}\delta_k \ln\sigma_{t-k}^2 + \sum_{k=1}^{q}\gamma_k\left[\left|z_{t-k}\right| - E\left(\left|z_{t-k}\right|\right)\right] + \sum_{k=1}^{q}L_k z_{t-k} ,$$

where have been introduced standardized (with unit variance) investigated indicators $z_{t-k} = \varepsilon_{t-k}/\sigma_{t-k}$, at that $E\left(\left|z_{t-k}\right|\right) = \sqrt{2/\pi}$ for the normal distribution of standardized indicators. Restrictions on the parameters of the model are due to the fact that all the roots of the characteristic equation:

$$\lambda^P - \delta_1\lambda^{p-1} - \delta_2\lambda^{p-2} - \ldots - \delta_p = 0$$

have been within the circle of radius.

For forecasting purposes, the GARCH model is usually complemented by any model that describes the behavior of the conditional or unconditional average number of observations. For example, it can be assumed that there is not ε_t, but ε_t plus a constant, that is, the observed series has a constant unconditional expectation β, to which is added a bug ε_t in a GARCH process: $y_t = \beta + \varepsilon_t$. It can be simulated as the unconditional mathematical expectation using linear regression, that is $y_t = X_t\alpha + \varepsilon_t$. This allows considering linear trend, deterministic seasonal variables, etc.

3. Results of the analysis

Estimation of ARCH model's parameters is held by the method of maximum probability and implemented in many application packages of statistical data. The econometric time-series analysis was conducted using the EViews 6 package in this study.

The first stage of the study is to identify the ARCH-effect. ARCH-test allows making conclusions about the volatility clustering, the presence of volatile periods and periods of relative quiet, which is to test the hypothesis that occasional errors of residues described by the ARCH model.

After examining a number of dynamic volumes of portfolio investments in Ukraine for the period from I quarter 1998 till IV quarter 2015 (Figure 1a), it is found that the ARCH effects have the first distinction of the series (Figure 1b − fluctuations for the period from I quarter 1998 till I quarter 2004 − low, from IV quarter 2004 till IV quarter 2015 − High; Figure 1c − statistics value $\chi_p^2 = 0{,}0123$ is less than 0.05, which means that − there exists the ARCH effect).

Calculate parameters of following econometric models ARCH family: ARCH (4), ARCH (5), GARCH (1,1), TARCH (1,1,1) and EGARCH (1,1,1). So:

ARCH(4)

$$\sigma_t^2 = 1253672 + 0{,}232\varepsilon_{t-1}^2 - 0{,}079\varepsilon_{t-2}^2 - 0{,}024\varepsilon_{t-3}^2 + 0{,}217\varepsilon_{t-4}^2,\ \beta = 25{,}743.$$

ARCH(5)

$$\sigma_t^2 = 1310153 + 0{,}403\varepsilon_{t-1}^2 - 0{,}121\varepsilon_{t-2}^2 - 0{,}045\varepsilon_{t-3}^2 + 0{,}290\varepsilon_{t-4}^2 + 0{,}037\varepsilon_{t-5}^2,\ \beta = -46{,}390$$

GARCH(1,1)

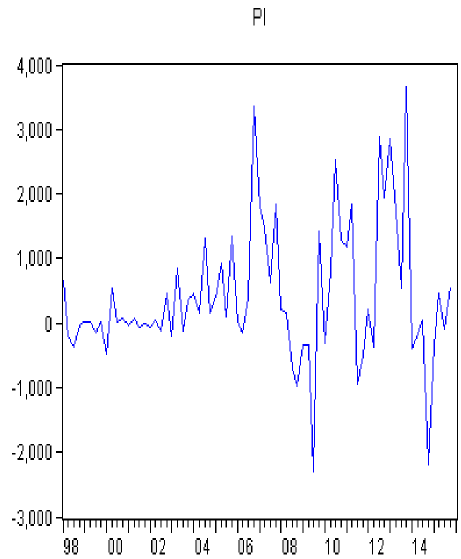
$$\sigma_t^2 = 1072263 + 0{,}526\varepsilon_{t-1}^2 - 0{,}117\sigma_{t-1}^2,\ \beta = 9{,}458$$

TARCH(1,1,1)

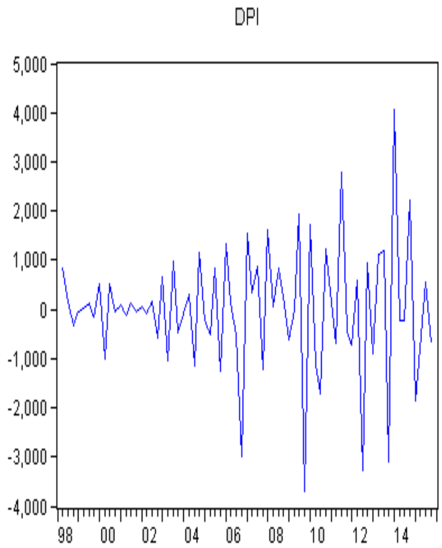
$$\sigma_t^2 = 949034{,}2 + 0{,}697\varepsilon_{t-1}^2 + 0{,}291\varepsilon_{t-1}^2(\varepsilon_{t-1} < 0) - 0{,}111\sigma_{t-1}^2,\ \beta = -84{,}715$$

EGARCH(1,1,1)

$$\log(\sigma_t^2) = 12{,}886 + 1{,}175\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - 0{,}177\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + 0{,}023\ln(\sigma_{t-1}^2),\ \beta = -52{,}001.$$



a)



b)

Heteroskedasticity Test: ARCH

F-statistic	6.743206	Prob. F(1,61)	0.0118
Obs*R-squared	6.271064	Prob. Chi-Square(1)	0.0123

c)

Fig. 1. a) Chart of portfolio investments; b) Chart of first distinctions; c) the results of research using ARCH-test

Source: own study.

To compare the quality of models, use Akaike information criterions (AIC) and Schwarz (the SCI) (Table 1). The lower criterion value, the higher the relative quality of the model.

Table 1. Akaike and Schwartz value criterion

Model	AIC	SCI
ARCH(4)	16,79	16,99
ARCH(5)	17,16	17,40
GARCH(1,1)	17,09	17,23
TARCH(1,1,1)	17,12	17,29
EGARCH(1,1,1)	17,05	17,22

Source: own study.

The table shows that the best model is the GARCH (4), after it is GARCH (1,1).
Checking ARCH(4) model for adequacy do by the following algorithm:

1. 1. Autocorrelation coefficients of residues for ARCH (4) model are shown in Figure 2. They are small, located in the vicinity of zero in the range. $\pm 2/\sqrt{n}$.

In addition, check autocorrelation coefficients of residues through test χ^2 ("x-square") which is based on *Ljung-Box Q statistic*. This test evaluates the overall sizes of the autocorrelation coefficients of residues. If the value Prob, related to *Q statistic* small (Prob), then there is autocorrelation. Correlogram shown in Figure 2 indicates that the autocorrelation in the model residues is absent.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.110	0.110	0.8180	0.366
		2	-0.129	-0.142	1.9432	0.378
		3	-0.076	-0.046	2.3462	0.504
		4	-0.144	-0.152	3.8020	0.433
		5	-0.147	-0.137	5.3394	0.376
		6	-0.161	-0.191	7.2165	0.301
		7	0.002	-0.035	7.2167	0.407
		8	0.039	-0.063	7.3298	0.502
		9	0.010	-0.069	7.3372	0.602
		10	-0.018	-0.107	7.3626	0.691
		11	0.223	0.184	11.309	0.418
		12	0.147	0.068	13.058	0.365
		13	-0.089	-0.062	13.710	0.395
		14	-0.010	0.052	13.719	0.471
		15	-0.076	-0.044	14.217	0.509
		16	0.108	0.218	15.241	0.507
		17	0.086	0.151	15.907	0.530
		18	-0.025	0.052	15.964	0.595
		19	0.030	0.084	16.048	0.654
		20	-0.064	0.007	16.443	0.689
		21	0.000	0.144	16.443	0.744
		22	-0.136	-0.118	18.294	0.688
		23	-0.027	-0.013	18.371	0.737
		24	-0.124	-0.209	20.003	0.697
		25	0.026	0.015	20.075	0.743
		26	0.153	0.074	22.664	0.652
		27	0.158	0.071	25.511	0.546
		28	0.171	0.009	28.961	0.414

Fig. 2. Correlogram for squares of standardized residues

Source: own study.

2. 2. Check existence of ARCH-effects in residues (Figure 3). Value of statistics $\chi_p^2 = 0{,}3784$, which is more than 0.05, which means – there is no ARCH effect.

Heteroskedasticity Test: ARCH

F-statistic	0.760498	Prob. F(1,61)	0.3866
Obs*R-squared	0.775760	Prob. Chi-Square(1)	0.3784

Fig. 3. The results of research using ARCH-test

Source: own study.

Thereby, the model adequately reflects residues.

Figure 4 shows the real data of portfolio investments and forecast values ARCH4 model.

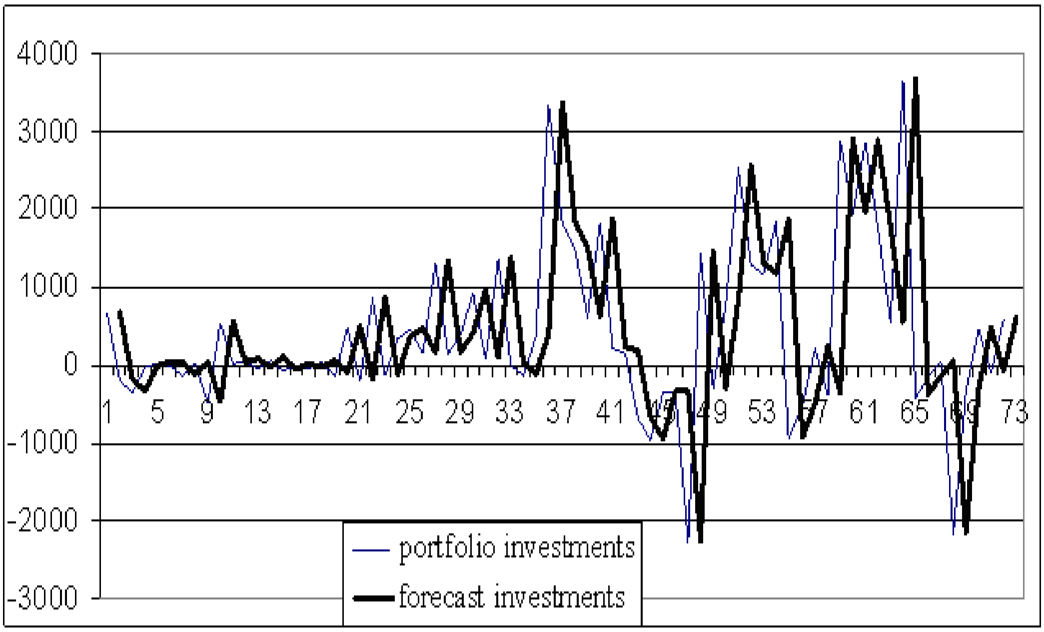


Fig. 4. Actual and forecast volumes of portfolio investments in Ukraine

Source: own study.

4. Conclusion

Taking into account that the focus of portfolio investments is on financial tools, and considering the characteristics of them, we consider that it is appropriate for forecasting to apply the ARCH model family. According to the outcomes of our research, this makes it possible to obtain good results. Forecasting the volatility of portfolio investments may have practical significance with respect to their management and in terms of the theoretical expansion of foreign investment forecasting methods.

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