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SOME ASPECTS OF SECURITY PORTFOLIO OPTIMIZATION

Summary: The existing economic and mathematical patterns of security portfolio optimization are supplemented with the correlation between investment shares. The problem of capital saving, which is urgent for marketable securities portfolio, is considered. The structure of the security portfolio the expected rate of return of which would be equal to the prescribed value and the risk rate of which would be minimal is set. A compromise alternative, which takes into consideration the expected rate of return as well as he risk, is considered.

Key words: the problem of capital saving, optimization, the expected rate of return, the risk rate of security portfolio

1. Introduction

With the introduction of economic reforms in Ukraine, operations with securities have become an integral part of the business activity of enterprises and firms. Securities play an important role in state payment operations as well as in the mobilization of capital investments. Securities enable capital transfer from investors to manufacturers and make it possible to determine the efficiency of financial assets usage in different sectors of economy.

In the developed countries under the conditions of balanced economy, a considerable part of the disposable capital is invested directly into security purchase.

With the aim of reducing the risk rate, securities are combined into portfolios. Harry Markowitz is the founder of the modern theory of portfolios, who was awarded Nobel Prize in Economic Sciences for his investigations in 1990. The detailed analysis of the investigations in this sphere is offered in the following article [1].

An essential contribution into the development of economic and mathematical modeling of security portfolio has been made by V. Vitlinskyi and his followers [2; 3]. Some of the problems and features of securities are analyzed in the following works [4; 5; 6].

The aim of the article is the development of the economic and mathematical modeling of the security portfolio, taking into consideration the correlation between security shares in the portfolio as well as creating a compromise alternative.

2. Security portfolio

Security portfolio is a set of different types of securities, bought by an investor with the aim of making a profit, which corresponds to certain requirements as to the admissible risk and profit rates.

The portfolio may include securities of the same type as well as various financial values (shares, obligations, savings certificates, etc.).

The structure of the portfolio is the correlation of certain types of securities in the portfolio. Building a portfolio, the investor is guided by the wish to have funds in such a form and place so that they could be secure, freely available and bring high profit.

The following factors should be taken into consideration while building a portfolio:

- the risk rate;
- profitability;
- investment term;
- security type.

The content and the type of the security portfolio depend not only on the investment policy, but also on the aim that has been set in the process of its building and management. The main aim of the process of security portfolio management may be defined in a different way. Most frequently, a security portfolio is built with the aim of:

- income maximization;
- the maintenance of liquidity;
- decreasing the risk rate.

While choosing the aim, priorities should be set out, as far as simultaneous fulfillment of all the objectives is impossible in practice. Although, a management team can make a decision to subdivide the portfolio into several parts, each of them contributing to the main aim. In our opinion, the chief aim of building security portfolios is the minimization of the risk rate to a certain level and, at the same time, providing a proper profitability. Nevertheless, the notion of a proper profitability and acceptable risk rate are not universal and depend on the chosen market behavior strategy, the type of business, the importance of financial investments in the total assets. Hence, the structure of the portfolio is set individually, by each enterprise, depending on the aims of its functioning.

3. The task of capital saving

Let us look closer at the task of capital saving. This task is actual for the portfolio with marketable securities. The economic and mathematical model is [3] as follows:

$$V_n = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \to \min$$

$$\sum_{i=1}^{n} x_i = 1, \tag{1}$$

$$x_i \geq 0, \quad i = \overline{1;n}.$$

Here, V_n is the security portfolio risk rate as the dispersion of the profit norm, which is determined according to the formula:

$$V_n = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij},$$

x_i — the investment share into securities of i-type ($0 \leq x_i \leq 1$),

$$\sigma_{ij} = \sigma_i \cdot \sigma_j \cdot \rho_{ij},$$

σ_i — the average quadratic deviation from the profit level (risk) of the securities of i-type, ρ_{ij} — the factor of correlation between the securities of the i- and j-type, n — the number of security types ($n \geq 2$).

We suggest supplementing the basic model (1) with the following correlation between investment shares $\sum_{i=1}^{n} k_i x_i + c = 0$, k_i, c — constant values.

For solving task (1) with the supplement, Lagrange multiplier method [3] is used. The following functional relation should be built:

$$L = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_i \sigma_j \rho_{ij} + \lambda_1 \left(\sum_{i=1}^{n} x_i - 1 \right) + \lambda_2 \left(\sum_{i=1}^{n} k_i x_i + c \right).$$

Let us find the functional relationship minimum values. With this aim, partial derivatives of the first rate of functional relation L are equaled to zero with regard to the values x_i, λ_1, λ_2.

A system $(n + 2)$ of linear algebraic equation with the same amount of unknown values will be obtained. It should be noted, though, that Lagrange method does not take into consideration the condition

$x_i \geq 0$. If all the solutions are nonnegative, then the task is solved. If some of the x_i are negative, the smallest value should be picked out and considered to be equal to zero. Then, the optimization problem is solved without a security of the corresponding type. The process of excluding "unfavourable" securities should be carried out up to the moment when the rest of the nonnegative values of x_i are obtained.

Example 1. The securities of four types are σ_1=10%, σ_2=15%, σ_3=20%, σ_4=25% and correlation coefficient of the profit norms is $\rho_{13} = –0,3; \rho_{12} = \rho_{14} = \rho_{24} = 0,2; \rho_{23} = \rho_{34} = –0,4$. Define security shares in the portfolio with the minimum risk rate if $x_1 = 2x_2$.

The formula for defining the risk rate of the portfolio looks as follows

$$V_n = 100x_1^2 + 60x_1x_2 - 120x_1x_3 + 100x_1x_4 + 225x_2^2 - 240x_2x_3 + 150x_2x_4 + 400x_3^2 - 400x_3x_4 + 625x_4^2.$$

As a result, economic and mathematical model has the following form:

$$V_n = 100x_1^2 + 60x_1x_2 - 120x_1x_3 + 100x_1x_4 + 225x_2^2 - 240x_2x_3 + 150x_2x_4 + 400x_3^2 - 400x_3x_4 + 625x_4^2 \to \min,$$

$$x_1 = 2x_2,$$

$$x_1 + x_2 + x_3 + x_4 = 1,$$

$$x_i \geq 0, \quad i = \overline{1;4}.$$

The following solution will be obtained: $x_1 = 0,4332$; $x_2 = 0,2166$; $x_3 = 0,2658$; $x_4 = 0,0844$, $V_n^{\min} = 37,4544$.

Without taking into consideration the condition $x_1 = 2x_2$, the following results will be obtained: $x_1 = 0,4251$; $x_2 = 0,2238$; $x_3 = 0,2667$; $x_4 = 0,0844$, $V_n^{\min} = 37,4396$.

Setting a supplementary condition results in the increase of the risk rate of the portfolio.

Let us investigate the process of receiving the planned profit. The main difficulty is to define the structure of the security portfolio, so that the expected profit level of the portfolio could be equal to the value m_c, while the risk rate could be minimal.

Economic and mathematical model looks as follows [3].

$$\begin{gathered} V_n \to \min \\ \sum_{i=1}^{n} x_i m_i = m_c, \\ \sum_{i=1}^{n} x_i = 1, \\ x_i \geq 0, \quad i = \overline{1;n}. \end{gathered} \tag{2}$$

In this model, m_i is the expected norm of the profit from the securities of i-type.

Let us supplement model (2) with the correlation between the investments shares provided earlier. Lagrangian functional relation will be as follows:

$$L = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_i \sigma_j \rho_{ij} + \lambda_1\left(\sum_{i=1}^{n} x_i - 1\right) + \lambda_2\left(\sum_{i=1}^{n} k_i x_i + c\right) + \lambda_3\left(\sum_{i=1}^{n} x_i m_i - m_c\right).$$

Example 2. Let us assume that, in addition to the information provided in example 1, the following values are known: $m_1 = 20\%$, $m_2 = m_3 = m_c = 30\%$, $m_4 = 50\%$. Define the security share in the portfolio with minimum risk rate if $x_1 = x_2$.

The mathematical model will look as:

$$V_n = 100x_1^2 + 60x_1x_2 - 120x_1x_3 + 100x_1x_4 + 225x_2^2 - 240x_2x_3 + 150x_2x_4 + 400x_3^2 - 400x_3x_4 + 625x_4^2 \to \min,$$

$$20x_1 + 30x_2 + 30x_3 + 50x_4 = 30,$$

$$x_1 = x_2,$$

$$x_1 + x_2 + x_3 + x_4 = 1,$$

$$x_i \geq 0, \quad i = \overline{1;4}.$$

In this case, Lagrange functional relation will look as

$$L = 100x_1^2 + 60x_1x_2 - 120x_1x_3 + 100x_1x_4 + 225x_2^2 - 240x_2x_3 + 150x_2x_4 + 400x_3^2 - 400x_3x_4 + 625x_4^2 +$$
$$+\lambda_1(x_1 + x_2 + x_3 + x_4 - 1) + \lambda_2(x_1 - x_2) + \lambda_3(20x_1 + 30x_2 + 30x_3 + 50x_4 - 30).$$

Partial derivatives of Lagrange functional relation are as follows:

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 200x_1 + 60x_2 - 120x_3 + 100x_4 + \lambda_1 + \lambda_2 + 20\lambda_3, \\ \dfrac{\partial L}{\partial x_2} = 60x_1 + 450x_2 - 240x_3 + 150x_4 + \lambda_1 - \lambda_2 + 30\lambda_3, \\ \dfrac{\partial L}{\partial x_3} = -120x_1 - 240x_2 + 800x_3 - 400x_4 + \lambda_1 + 30\lambda_3, \\ \dfrac{\partial L}{\partial x_4} = 100x_1 + 150x_2 - 400x_3 + 1250x_4 + \lambda_1 + 50\lambda_3, \\ \dfrac{\partial L}{\partial \lambda_1} = x_1 + x_2 + x_3 + x_4 - 1, \\ \dfrac{\partial L}{\partial \lambda_2} = x_1 - x_2, \\ \dfrac{\partial L}{\partial \lambda_3} = 20x_1 + 30x_2 + 30x_3 + 50x_4 - 30. \end{cases}$$

Having equaled them to zero and having solved the system, we will obtain the following solution:

$$x_1 = x_2 = 0{,}2803;\ x_3 = 0{,}2992;\ x_4 = 0{,}1402;\ V_n = 41{,}1956.$$

Without taking into consideration the condition $x_1 = x_2$, the values of the shares are: $x_1 = 0{,}3010$; $x_2 = 0{,}2506$; $x_3 = 0{,}2979$; $x_4 = 0{,}1505$; $V_n = 40{,}9466$.

Let us define a compromise alternative [7] on the example of the two problems. The first problem is example 1 without the condition $x_1 = 2x_2$. The second problem looks as follows:

$$m_n = 20x_1 + 30x_2 + 30x_3 + 50x_4 \ \rightarrow \max,$$
$$\sum_{i=1}^{4} x_i = 1,$$
$$x_i \geq 0, \quad i = \overline{1;4}.$$

Here m_n is the expected norm of the security portfolio profit. The values m_i $\left(i = \overline{1;4}\right)$ are taken from example 2.

The solution to this problem is as follows: $x_1 = x_2 = x_3 = 0$; $x_4 = 1$; $m_n^{\max} = 50$. Having included the received values in the equation for V_n in example 1, we will obtain $V_n = 625$.

The general form of the compromise alternative is as follows:

$$z = x_5 \ \rightarrow \ \min,$$
$$V_n - V_n^{\min} \leq V_n^{\min} \cdot x_5,$$
$$m_n^{\max} - m_n \leq m_n^{\max} \cdot x_5,$$
$$\sum_{i=1}^{4} x_i = 1,$$
$$x_i \geq 0, \quad i = \overline{1;5}.$$

Having included formulas for V_n in example 1 and m_n from the second problem, we will have

$$z = x_5 \ \rightarrow \ \min,$$
$$100x_1^2 + 60x_1x_2 - 120x_1x_3 + 100x_1x_4 + 225x_2^2 - 240x_2x_3 + 150x_2x_4 + 400x_3^2 -$$
$$-400x_3x_4 + 625x_4^2 - 37{,}4396x_5 \leq 37{,}4396,$$
$$2x_1 + 3x_2 + 3x_3 + 5x_4 + 5x_5 \geq 5,$$

$$x_1 + x_2 + x_3 + x_4 = 1,$$
$$x_i \geq 0, \quad i = \overline{1;5}.$$

The solution is x_1 = 0,7088; x_2 = 0,2491; x_3 = 0; x_4 = 0,0421; x_5 = 1,1491. Then m_n = 23,7540; V_n = 80,4597. In comparison to 50 as the maximum values of profit and 625 as risk rate, we will observe their essential decrease, especially the decrease of the risk rate.

4. Conclusions

Our succeeding investigations will be devoted to the problems of obtaining capital gains using a security portfolio.

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NIEKTÓRE ASPEKTY OPTYMALIZACJI PORTFELA PAPIERÓW WARTOŚCIOWYCH

Streszczenie: znane ekonomiczno-matematyczne modele optymalizacji portfela papierów wartościowych są uzupełnione współzależnością między częściami inwestycji. Rozpatrzono zadanie zachowania kapitału, które jest istotne dla portfela płynnych papierów wartościowych. Określono strukturę portfela papierów wartościowych, oczekiwana stopa zwrotu którego będzie równa zadanej wielkości, a ryzyko byłoby najmniejsze. Rozpatrzono kompromisowy wariant, który uwzględnia spodziewaną stopę dochodu i ryzyko.

Słowa kluczowe: zadanie zachowania kapitału, optymalizacja, oczekiwana stopa zwrotu, ryzyko portfela papierów wartościowych