MODELLING OF THE HEAT FLOWS DISTRIBUTION IN ENGINEERING NETWORKS

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Heat supply engineering networks are examined in paper. Mathematical statement of equal heat flows distribution problem is stated. Graph is used for modelling of the heat flows distribution in heat supply networks. Original algorithm is offered for the solution of the heat flows distribution problem.

Engineering systems comprise different pipe, ventilation and electric systems. Each of them consists of three subsystems. They are the source of target commodity, processing subsystem, transport and distribution subsystem. The first and the second subsystems prepare the target commodity for consumption. The last one, which is called engineering network, distributes the target commodity. Heat is target commodity in heat supply networks.

To deliver equal heat amount to the consumers is the topical problem of the operation of heat supply systems. Effective use of heat supply networks reduces material and power costs and improves heat supply of consumers.

It is possible to set the problem of equal heat distribution as constraint optimization problem. It is necessary to ensure minimum deviations of quantity of heat consumption from the average heat amount. The pipes' capacities and the wastage of heat are limited. The rule of flow saving in node is right for the heat supply network also.

Graph model is favourable for modelling of the heat flows distribution in heat supply networks. Consider the heat supply network of Figure 1.

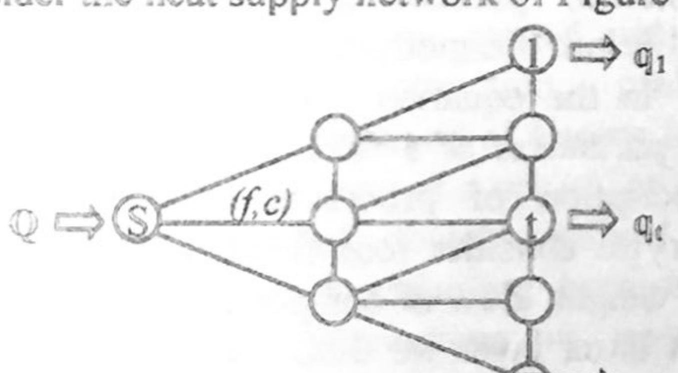

Figure 1. The graph model of heat supply network

In Figure 1 S is the heat station (heat source); the nodes are the heat distribution centres; T is the subset of heat consumers (heat sinks); the lines are the pipes; c is the capacity of pipe; f is the heat flow in pipe.

Now we may use the node-arc formulation of the problem. It may be stated as follows:

Minimise $$D=\sum_{t=1}^{T}\left[q_t-\frac{Q}{T}\right]^2 \qquad (1)$$

Subject to

$$0\le f_k\le c_k,\quad k\in\overline{1,M}, \qquad (2)$$

$$\sum_{k\in M_{Oi}} f_k-\sum_{k\in M_{Pi}} f_k=0,\quad i\in N-\{S,T\}, \qquad (3)$$

$$\sum_{k\in M_{Oi}} f_k-\sum_{k\in M_{Pi}} f_k-Q=0, \qquad (4)$$

$$\sum_{k\in M_{Oi}} f_k-\sum_{k\in M_{Pi}} f_k+q_t=0,\quad t\in\overline{1,T}, \qquad (5)$$

$$\sum_{t=1}^{T} q_t=Q, \qquad (6)$$

where Q is the total heat amount; q_t is the heat amount of each consumer; T is a number of consumers; M is a number of arcs in graph; N is a number of nodes in graph; O is the list of initial nodes of arcs; P is the list of ending nodes of arcs; M_{Oi} is the list of arcs which leave the node i; M_{Pi} is the list of arcs which come in node i.

To solve the problem of equal heat distribution we may use the maximal flow problem. The maximal flow problem is the flow programming problem. It may be stated as follows. Consider a network with N nodes and M arcs through which a single commodity will flow. Arc capacities are specified. We consider that the rule of flow saving in node is right for the network also. In such a network, we wish to find the maximum amount of flow from the source (node 1) to the sink (node N).

As you see the statement of the problem of equal heat flows distribution and the statement of maximal flow problem are similar.

Ford and Fulkerson have offered an algorithm to solve the maximal flow problem [1]. This algorithm may be stated as follows. First select a set of feasible flows, flows' values usually are zero. Then attempt to locate a path from source to sink. Add Δ to flows on arcs, where Δ is the minimum difference among capacities and flows of the associated chain. Attempt to locate new path from source to sink. The optimal solution will be obtained if no such path exists.

To solve the problem of equal heat flows distribution we offer to transform the graph of heat supply network (Figure 2). We add dummy source node S_0 and dummy arc with capacity C^{max}, where C^{max} is the maximal heat amount, which the heat station may produce. We accept that the maximal heat flow, which may pass through the network, always overpasses produced heat amount. The sink nodes we link to dummy sink T_0.

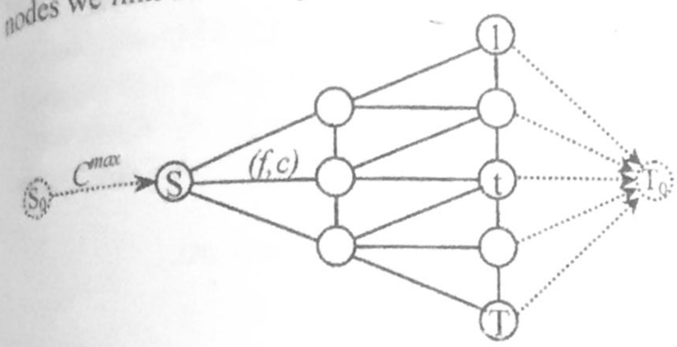

Figure 2. Transformed graph.

The maximal flow problem may be solved on graph presented by Figure 2.

While solving the problem of equal heat flows distribution following cases are feasible.

1) The structure of network allows the equal heat flows distribution. This case is the simplest and we may solve our problem as maximal flow problem. It's enough to limit the capacities of dummy arcs, which link the heat sinks and the dummy sink T_0. This limit capacity is $C_t^{max} = Q/T$.

2) The structure of network allows to distribute the maximal heat flow equally. The real heat flow Q is less than $T * \min(q_1^{max},...,q_T^{max})$, where $\min(q_1^{max},...,q_T^{max})$ is the minimum of maximal heat flows, which pass over the heat sinks. In this case we may solve our problem as maximal flow problem. It's enough to limit the capacities of dummy arcs, which link the heat sinks to the dummy sink T_0. This limit capacity is $C_t^{max} = \min(q_1^{max},...q_T^{max})$. But it is necessary to find T maximal flows to define C_t^{max}. Furthermore if minimal cut-sets for some maximal flows are equal then $C_t^{max} = \min(q_1^{max},...,q_T^{max})/d$, where d is a number of equal minimal cut-sets. That's why it is difficult to define C_t^{max} and our problem can't be solved as maximal flow problem.

3) The structure of network doesn't allow the equal heat flows distribution. The real heat flow Q is more than $T * \min(q_1^{max},...,q_T^{max})$. In this case it is impossible to solve the problem of equal heat flows distribution as maximal flow problem.

Three cases were examined and we may infer that it is impossible to solve the problem of equal heat flows distribution as maximal flow problem.

We offer an original algorithm to solve the problem of equal heat flows distribution, which associates all examined cases. The algorithm is following.

1. Add the dummy source S_0 and the dummy arc with capacity C^{max}. The sink nodes link to dummy sink T_0, the dummy arcs are not limited.
2. Set the arc capacities.
3. Use the Ford-Fulkerson algorithm to find the maximal heat flow Q from dummy source S_0 to dummy sink T_0.
4. Remove all dummy arcs. Define heat flows $q_1, q_2, ..., q_T$, which pass over the heat sinks.
5. Find two sink nodes T^{max} and T^{min} with q^{max} and q^{min} heat flows, where q^{max} and q^{min} are the maximum and the minimum heat flows accordingly.
6. If $q^{min} = q^{max}$ then go to step 13, otherwise go to step 7.
7. Accept T^{max} as heat source and T^{min} as heat sink. Locate pass from source to sink.
8. Define the minimal difference *dmax* among the capacities and the flows of the associated chain. If *dmax* is more than $(q^{max} - q^{min})/2$ then *dmax* set to $(q^{max} - q^{min})/2$. Add *dmax* to the flows on the arcs of associated chain.
9. Locate new pass from source to sink and add new *dmax* to the flows on the arcs of associated chain.
10. If no pass existed but flows were added then go to step 5, otherwise go to step 11.
11. Remove node T^{max} from the set of possible heat sources. If set of sources is not empty than go to step 5, otherwise go to step 12.
12. If set of sources is empty than remove T^{min} from the set of sinks and from the set of sources. If set of sinks is empty then go to step 13, otherwise go to step 5.
13. End.

Using the algorithm it is possible to find the optimal solution even in the case of several heat sources. Method may reduce the wastage of heat and distribute heat more efficiently in the case of impossibility of the equal heat distribution.

Software was designed too [2]. It may be used practically in CAD systems of heat flows distribution in heat supply engineering networks.

1. Ford L.R. and D.R. Fulkerson. *Flows in Networks*. Princeton, N.J., 1962.

2. *Паночишин Ю.М., Дубовой В.М.* Комп'ютерна програма оптимізації розподілу тепла Flow Pro / Свідоцтво про державну реєстрацію прав автора на твір № 3860, 5 лютого 2001 р.