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MAXIMIZING INVENTOR'S PROFIT IN THE PROCESS OF INVENTION COMMERCIALIZATION BASED ON THE NORDHAUS MODEL

Abstract

In this article, the author develops a model of inventor's profit maximization in the process of commercializing an invention based on the critical analysis of the «optimal life of a patent» model offered by the American economist William Nordhaus. The author defines perspective directions for further modifications of the Nordhaus model for its application in the real economic conditions of Ukraine.

Key words:

Nordhaus model, patent, invention, time lag, profit maximization, commercialization of an invention.

JEL: O31; O38.

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The Problem

The present times require science to develop practical instruments of decision-making in the process of managing innovation activity at all levels. In view of this, and in complete absence of national contributions, it is feasible to use in theoretical research and practice the advanced achievements of the leading foreign scientists in this field.

The model of the optimal life of a patent, offered by the American economist William Nordhaus back in 1967 [1], also known as the 'Nordhaus model', is so far largely unfamiliar to scientific circles of Ukraine, although the outcomes of this elaboration will remain urgent for our country, which declares its orientation at innovation-based development, for a long time to come.

When developing his model of the optimal life of a patent, William Nordhaus posed the problem of defining the conditions and the duration of patent life for different types of inventions, which together would produce maximal economic return for both the inventor and the economy and society as a whole.

The actuality of the above-mentioned model modification is proven by the fact that already in his recent research (2002), William Nordhaus has shown, based on the new methods of industrial productivity measurement, that in 1996-1998 labour productivity in both new (computer) industry and sectors of «old» economy has been growing faster than in the period of 1977–1995 [2], which led the scientist to conclude that the productivity in the whole US economy has been growing. The mentioned work asserts that this conclusion has found support in the report of the US Council of Economic Advisers, which acknowledged that the accelerated productivity growth had been taking place both in computer and other sectors of economy. W. Nordhaus found that the changes in labour productivity had cyclical and structural components, whereas the structural growth of labour productivity consisted of the following four factors:

- increase in capital;
- increase in the quality of labour force;
- technological progress in computer industry;
- technological progress in other branches.

Evidently, it is the innovation-based economic development of nations that provides for the availability of the two last components of structural labour productivity growth. At the same time, the realization of this way of development stipulates for continuous introduction of the leading achievements in science and technology to production.

In the paper [3], we for the fist time introduce the modification of the first part of the Nordhaus' optimal life of a patent model for the case of various economic growth rates, shifts in demand, volume of production, and presence of a time lag between the introduction of invention and the real economic return. The model is expanded by removing the explicit assumptions about constant discount rates and zero relative growth.

In this paper we shall focus on the first part of the Nordhaus model, which deals with inventions that allow to decrease the costs of production, or the so called 'regular' or 'run-of-the-mill' inventions. The introduction of these very inventions permits to increase labour productivity and reduce production costs, and simultaneously improve technological processes without heavy capital investment, since the Ukrainian economy is somewhat «wasteful».

Objective of the Research

In this paper we aim to reveal further directions for modification of the Nordhaus model in order to make it applicable to modern economic realities, and to expand the given model by means of mathematical tools, that is to develop the model of inventor's profit maximization in the process of invention commercialization.

The Results

The Nordhaus model being analyzed implicitly contains another assumption which has not been considered earlier in [3]. The fact is that the economic effect from introduction of an invention does not come to an inventor or a patent owner at once, when the output is produced at a lower cost, especially if the inventor is remunerated in the form of a royalty, since the final product first needs to be sold and paid, and only after that will the inventor receive his royalty. Even if we assume that the accounts receivable do not turn bad, which is not mentioned in the model, it is quite possible that a considerable time lag appears between the production of a good based on the invention introduced and the collection of remuneration for it.

Hence, we need to investigate the economic and mathematical problem of inventor's profit maximization taking into consideration the time lag between the implementation of new, invention-based technological processes and the receipt of remuneration.

A fragment of the Nordhaus model is devoted to maximizing the profit of an inventor, wherein the functional dependence is assumed to exist between value B and R&D costs as

$$B = \beta R^{\alpha}, \qquad (1)$$

$$\beta > 0, \quad 0 < \alpha < 1.$$

where B are savings on production costs after the introduction of innovative technology or an invention (Invention Possibility Function, IPF [4: 77]), β is the scale coefficient, R – R&D costs, and α is the R&D productivity coefficient.

In this case, the profit of an inventor is given by

$$\Pi = V - R. \tag{2}$$

where V is the royalty.

Under the constant absolute production growth, the profit of an inventor can be expressed on the basis of (1) and (2) and taking into account that $V = \frac{Bx_0}{r} - \frac{B(x_0 + x_1T)}{r}e^{-rT} + \frac{Bx_1}{r^2}(1 - e^{-rT})$, where V is the royalty-type return at x_1 – certain output after the invention, x_0 is the output before the invention, and r is the discount rate:

$$\Pi = \beta R^{\alpha}\left(\frac{x_0}{r} - \frac{x_0 + x_1T}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right) - R. \tag{3}$$

Having found the partial derivative of (3) for R, we will get

$$\frac{\partial \Pi}{\partial R} = \beta\alpha R^{\alpha-1}\left(\frac{x_0}{r} - \frac{x_0 + x_1T}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right) - 1. \tag{4}$$

Setting the speed of inventor's profit change relative to costs equal to zero, we get

$$\beta\alpha R^{\alpha-1}\left(\frac{x_0}{r} - \frac{x_0 + x_1T}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right) - 1 = 0. \tag{5}$$

From (5) we make the costs of R&D explicit by the following transformations:

$$\beta\alpha R^{\alpha-1} = \left(\frac{x_0}{r} - \frac{x_0 + x_1T}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right)^{-1};$$

$$R^{\alpha-1} = \frac{r}{\beta\alpha}\left(x_0 - (x_0 + x_1T)e^{-rT} + \frac{x_1}{r}\left(1 - e^{-rT}\right)\right)^{-1}.$$

Consequently,

$$R = \left(\frac{r}{\beta\alpha}\right)^{\frac{1}{\alpha-1}}\left(x_0 - (x_0 + x_1T)e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right)^{\frac{1}{1-\alpha}}. \tag{6}$$

Costs (6) make profit (3) extremal, i. e. maximal, since the acceleration of profit change is negative:

$$\frac{\partial^2 \Pi}{\partial R^2} = \beta\alpha(\alpha-1)R^{\alpha-2}\left(\frac{x_0}{r} - \frac{(x_0 + x_1 T)}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right) < 0.$$

We calculate the possible maximal profit $\Pi_{\max}$ by putting the value got in (6) into (3):

$$\Pi_{\max} = \beta\left(\frac{r}{\beta\alpha}\right)^{\frac{1}{\alpha-1}}\left(x_0 - (x_0 + x_1 T)e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right)^{\frac{\alpha}{1-\alpha}} \times$$
$$\times\left(\frac{x_0}{r} - \frac{x_0 + x_1 T}{r}e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right) -$$
$$-\left(\frac{r}{\beta\alpha}\right)^{\frac{1}{\alpha-1}}\left(x_0 - (x_0 + x_1 T)e^{-rT} + \frac{x_1}{r^2}\left(1 - e^{-rT}\right)\right)^{\frac{1}{1-\alpha}}.$$

As we see from the received formula, the maximal possible profit of an inventor non-linearly depends on every parameter in the model. At that, the greater is the speed of absolute productivity growth, the larger is the profit of an inventor:

$$x_1' < x_1'' \Leftrightarrow \Pi'_{\max} < \Pi''_{\max}.$$

Now we will consider the case of constant relative production growth, when profit V is expressed as [3]

$$V = Bx_0(1 - e^{(\mu - r)T})/(r - \mu) .$$

Taking into account (1) i (2), we get

$$\Pi = \beta R^{\alpha} x_0\left(1 - e^{(\mu - r)T}\right)/(r - \mu) - R. \qquad (7)$$

The speed of profit change Π relative to R&D costs R in this case is:

$$\frac{\partial \Pi}{\partial R} = \beta\alpha R^{\alpha-1} x_0\left(1 - e^{(\mu - r)T}\right)/(r - \mu) - 1. \qquad (8)$$

Equating (8) to zero, we get

$$\beta\alpha R^{\alpha-1} x_0\left(1 - e^{(\mu - r)T}\right)/(r - \mu) - 1 = 0; \quad \beta\alpha R^{\alpha-1} x_0\left(1 - e^{(\mu - r)T}\right) = r - \mu;$$
$$R^{\alpha-1} = \frac{r - \mu}{\beta\alpha x_0\left(1 - e^{(\mu - r)T}\right)}; \quad R^{1-\alpha} = \frac{\beta\alpha x_0\left(1 - e^{(\mu - r)T}\right)}{r - \mu};$$

$$R=\left(\frac{\beta\alpha x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{1}{1-\alpha}}. \qquad (9)$$

Accordingly, the savings on the unit cost of production with regard to (1) are:

$$B=\beta\left(\frac{\beta\alpha x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{\alpha}{1-\alpha}},$$

or

$$B=\beta^{\frac{1}{1-\alpha}}\left(\frac{\alpha x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{\alpha}{1-\alpha}}. \qquad (10)$$

Taking into account (10) and (1), the profit of an inventor V is expressed as

$$V=\beta^{\frac{1}{1-\alpha}}\left(\frac{\alpha x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{\alpha}{1-\alpha}} x_0\left(1-e^{(\mu-r)T}\right)/(r-\mu),$$

or, having done the simplification, as

$$V=\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{\beta x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{1}{1-\alpha}}. \qquad (11)$$

Based on (2), (9) and (11), we determine the maximal possible profit:

$$\Pi_{\max}=\left(\alpha^{\frac{\alpha}{1-\alpha}}-\alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{\beta x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{1}{1-\alpha}}. \qquad (12)$$

Formula (12) can be used in case the relative speed of production efficiency change μ does not exceed the discount rate r, i. e. under the condition similar to the one when we get (11). In case of exceeding the value $\mu > r$ the formula (12) can also be used, since V in (12) has the same analytical expression as (11).

Let us consider the case when values μ and r tend to equality and calculate

$$\lim_{\mu\to r}\Pi_{max} = \lim_{\mu\to r}\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{\beta x_0\left(1-e^{(\mu-r)T}\right)}{r-\mu}\right)^{\frac{1}{1-\alpha}} = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)(\beta x_0 T)^{\frac{1}{1-\alpha}}. \quad (13)$$

Specifically, at $\alpha = 1/2$ the profit $\Pi_{max}\big|_{\mu=r}$ depends on the duration of patent life according to the parabolic law:

$$\Pi_{max}\big|_{\mu=r} = \frac{1}{4}\beta^2 x_0^2 T^2. \quad (14)$$

Formula (14) demonstrates that, in this case, the profit is proportionate not only to the squared length of patent life, but also to the squared production efficiency at the initial moment of introducing a cost-saving invention.

It is clear that such an outcome can be obtained only under condition of zero time lag, in particular, under zero time lag in receipt of the royalty.

We should also note that the delays in time of receiving the return on cost-saving inventions can be generated by both technical and other factors, including economic and environmental ones.

For example, according to the authors of the Internet-site www.truba.com.ua, the introduction of newly-invented heating pipes in polyurethane casing of the «pipe in a pipe» type into the practice of housing construction ensures the following:

- a three-fold decrease in insulation-borne heat losses;
- a nine-fold decrease in operating costs;
- a three-fold decrease in maintenance costs;
- a 1.3-fold reduction in capital expenditures in the construction industry.

Nevertheless, this invention is being very slowly introduced in the heating system, since the substitution of the new pipelines for the existing ones requires substantial capital and time expenditures. Moreover, it is not quite clear for whom of the heating services market participants should such an economy on costs be advantageous, since in Ukraine there is practically no competition among the producers of heating services. At the same time, perfect competition is one of the fundamental preconditions for application of the Nordhaus model, which is proven by the above-mentioned example.

Let us now consider the problem of maximizing the profit of an inventor considering the presence of a time lag in receiving the remuneration. For this purpose, we place function (1) into formula $V = e^{-r\Delta t}\left(\frac{Bx_0}{r} - \frac{B(x_0 + x_1 T)}{r}e^{-rT} + \frac{Bx_1}{r^2}(1-e^{-rT})\right)$ [3] and get the profit V from the following formula:

$$V = \beta R^{\alpha} e^{-r\Delta t}\left(\frac{x_0}{r} - \frac{(x_0 + x_1 T)}{r} e^{-rT} + \frac{x_1}{r^2}(1 - e^{-rT})\right). \qquad (15)$$

The maximal possible profit in this case will be:

$$\Pi_{\max} = \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)\left(\beta e^{-r\Delta t}\left(\frac{x_0}{r} - \frac{(x_0 + x_1 T)}{r} e^{-rT} + \frac{x_1}{r^2}(1 - e^{-rT})\right)\right)^{\frac{1}{1-\alpha}}. \qquad (16)$$

As seen from (16), an increase in the remuneration time lag leads to a decrease in profit:

$$\Delta t' < \Delta t'' \Leftrightarrow \Pi'_{\max} > \Pi''_{\max}.$$

This might be the factor that forces some inventors to use more complex than patenting legal constructs for protection of their inventions.

Thus, for example, according to the Internet site www.ln.com.ua, the chemical agent – anamegator «Super Gold Ozerol» is produced in Ukraine by the production enterprise Adioz, which holds the rights to the invention. At that, the chemical composition and technology of production of the named agent comprise the know-how of the enterprise. This legal construct provides an opportunity to use the invention for practically an unlimited time, since the current law does not foresee any time limits for know how's or official intellectual property rights.

From the above-described example comes out another assumption, which is implicit in the Nordhaus model: an inventor or an R&D organization that develops inventions for production renovation or modification can cooperate only with enterprises whose production process is known to the potential inventor. Truly, it is difficult to improve the mode of goods or services production at an object protected by such an object of intellectual property as know-how, i.e. when the information about the mode of production is confidential.

Let us consider one more constraint of the Nordhaus model related to the analytical form of the function describing dependence between the size of possible savings B on unit cost of production and the R&D costs R.

We should pay attention to the fact that function (1) monotonically grows when R grows, but that growth has no upper limit, that is when $R \to +\infty \Rightarrow B \to +\infty$. This means that function (1) can be defined only up to a certain amount of costs $R = R_{kp}$: since savings on costs can not tend to plus infinity, the size of saved costs should have a ceiling**,** be it at least the unit production cost, although some speak of a certain value D that by the assumption about standardized input costs should not exceed unity:

$$D < 1. \qquad (17)$$

Thus, it would be logical to assume that the function describing the dependence of the savings on production costs upon the R&D costs will be

$$B = D\left(1 - e^{-\alpha r}\right). \quad (18)$$

By using the tools of mathematical analysis, we can make sure that (18) retains two basic properties of (1), namely the monotonous growth and concavity (i.e. upward convexity), though it is deprived of the defect of infinite growth.

Placing (18) into the formula $V = \int_0^T Bx_0 e^{-rT} dt = \frac{Bx_0}{r}(e^{-rT})$ offered in [4: 77], we get the following expression for computation of the total financial profit of an inventor:

$$V = \frac{Dx_0}{r}\left(1 - e^{-\alpha r}\right)(1 - e^{-rT}). \quad (19)$$

Proceeding from (2) and (19), the profit of an inventor is:

$$\Pi = \frac{Dx_0}{r}\left(1 - e^{-\lambda r}\right)(1 - e^{-rT}) - R. \quad (20)$$

Let us find the volume of investments in R&D yielding maximal profit to the inventor:

$$\frac{\partial \Pi}{\partial R} = \frac{\lambda D x_0}{r} e^{-\lambda r}(1 - e^{-rT}) - 1; \frac{\lambda D x_0}{r} e^{-\lambda r}(1 - e^{-rT}) - 1 = 0 \Rightarrow$$

$$\Rightarrow e^{-\lambda r} = \frac{r}{\lambda D x_0 (1 - e^{-rT})} \Rightarrow$$

$$R = \frac{1}{\lambda} \ln\left(\frac{\lambda D x_0 (1 - e^{-rT})}{r}\right). \quad (21)$$

Consequently, the decision about R&D financing in volume (21), which derives from the assumption about functional dependence of expected savings on costs (18) requires no additional test of whether the size of financing exceeds the critical level, as it is the case in the classical version of the Nordhaus model.

Similarly, (18) could be applied when certain constraints of the initial version of the Nordhaus model need to be rejected. For example, in the case of several different discount rates as in (19) and the respective income *V* in (20), the optimal R&D costs are:

$$R = \frac{1}{\lambda} \ln\left(\lambda D x_0 \sum_{i=1}^{n} \left(e^{-r_i T_{i-1}} - e^{-r_i T_i}\right) / r_i\right). \quad (22)$$

Here we should note that the person, who in practice plans to finance R&D not as a business-angel, but as a profit-seeking individual, could be concerned with some questions other than the mere profit size. For example, if one chooses joint-stock financing, then the most important indicator for the potential stock or investment portfolio holders is the profitability of, or return on, a the given specific security. Exactly this circumstance has drawn attention of H. Markowitz, the founder of the modern portfolio theory.

Hence, based on (22) let us solve for costs the profitability of R&D, or more precisely, the present aggregate profitability:

$$\rho = \frac{\Pi}{R} = \frac{V - R}{R} = \frac{V}{R} - 1, \tag{23}$$

where profit V is calculated by combining (20) and (23); the expression thus looks as

$$\rho = \frac{Bx_0}{R} \sum_{i=1}^{n} \left(e^{-r_i T_{i-1}} - e^{-r_i T_i} \right) / \, r_i - 1. \tag{23.a}$$

or taking into account (18) as:

$$\rho = D\left(1 - e^{-\lambda R}\right) \frac{x_0}{R} \sum_{i=1}^{n} \left(e^{-r_i T_{i-1}} - e^{-r_i T_i} \right) / \, r_i - 1. \tag{23.b}$$

Formula (23b) could be used in at least two ways, either to calculate profitability at given costs R according to (22), which provides for the maximal profit, or to use it as a target function for maximizing the present aggregate profitability:

$$\rho \to \max. \tag{24}$$

Nevertheless, profitability ρ as the function of costs R in (23b) is a monotonic downward sloping function; therefore, in this case the problem of profit maximization (24) does not have a non-trivial solution. Similar conclusion is derived when function (1) offered by Nordhaus is used to calculate profitability.

Thus, here we should mention one more point, which equally concerns both functions (1) and (18), in particular their behaviour about a zero argument *R.* At zero *R*, functions (1) and (18) turn to zero, which proves the assumption that it is impossible to save on production costs without investing in R&D.

Nevertheless, even at minor investments under (1), savings on costs are possible. However, this assumption, probably, contradicts the essence of the Nordhaus model, according to which the potential patent holders have no other alternative to invent. Thus, even if minor investments are cost-saving, then why cannot competitors undertake similar investments in R&D? That is, the part of the model concerning the comparison of R&D costs between competitors needs further specification, since the presence of R&D costs by itself does not expose their efficiency in terms of innovation-based development.

Conclusions

We developed the model of inventor profit maximization in the process of invention commercialization based on the Nordhaus model, which allowed us to reveal the complex character of the individual's motives to engage in inventing as intellectual labour and, respectively, to undertake R&D activity. This permits us to ascertain that the sole probability of commercializing a certain invention depends on quite a few parameters of the innovation project, invention and the patent, as well as on legal instruments of intellectual property rights protection and their combinations. In addition, it should be indicated that exactly the factors of environment produce a great effect on the process of invention commercialization and the profit of an inventor. All this is breaking quite promising grounds for research, in particular the direction for developing methods and models of forming an effective environment for commercialization of inventions both at the national and supranational levels.

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