

Construction of Two-Dimensional Correlation Models in a Cartesian and Spherical Coordinate System

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Abstract: This paper presents a methodology for constructing and visualizing correlation models of two-dimensional signals in a rectangular and spherical coordinate system.

Keywords: correlation models, spherical coordinate system, two-dimensional models.

I. INTRODUCTION

The mathematical apparatus of correlation analysis in the Cartesian coordinate system remains a powerful tool for researching technological processes and natural phenomena [1]. Areas of its application are extremely diverse both in the fundamental directions of science and applied science. Correlation analysis is successfully used in the study of processes in atomic and nuclear physics, energy, electronics, radio engineering, astronomy, astrophysics, economics and other fields of science [2-4].

Correlation models give an opportunity to investigate both one-dimensional and multidimensional deterministic and stochastic processes [5]. It is clear that the correlation analysis is not universal, has its own limitations and possibilities of use for a certain class of tasks. However, this is a convenient and reliable method for solving a wide range of tasks, which needs further development and improvement for its greater efficiency and extending its use.

At the present stage of microprocessor development and computer technology, new perspectives of correlation models development are opened, which will allow them to be used in real-time systems and create more complex models.

The article presents mathematical expressions for constructing correlation models in a spherical coordinate system and proposes to take into account the effect of "aging" information and transport delays in correlation models. The construction of correlation models in a spherical coordinate system opens up new prospects for their use in the study of processes and phenomena, which are simpler described in spherical coordinates than in Cartesian space. Such processes include the determination of orbits and figures of celestial bodies, the determination of objects' movement in space, radar, and propagation of waves of various nature in space, trajectories of motion of elemental particles in atoms, rotating processes in mechanics, determination of precise coordinates in cartography, and many others.

The article presents the first theoretical stage of the study of the satellite's interconnection in a stationary orbit with points on the Earth's surface. It consists in the development of a mathematical apparatus for constructing a correlation model in a spherical coordinate system. For the formation of the model were adopted simplifications, which consist in taking

the form of the Earth in the form of a ball, the satellite's orbit is stationary in the form of a circle over a certain latitude of the Earth. To assess the adequacy of the model an analysis of its results over the elementary types of signals presented in the form of sinusoidal.

II. TWO-DIMENSIONAL PROCESSES IN CARTESIAN AND SPHERICAL COORDINATE SYSTEMS

The definition of correlation in a spherical coordinate system, by the way, as in the Cartesian or D-denier coordinate system, requires a clear definition whether it is necessary to make correlations of figures or correlation of processes.

In the first case, under the correlation of the figures it is necessary to understand the correlation of the coordinates of the points belonging to a bulk figure with the corresponding coordinates of the points of the standard figure. In this case, it is sufficiently to consider the functions describing the studied and standard figures given in parametric or normal form, while the functions depend on time, so the time coordinate is not taken into account. This means that spatial figures remain unchanged in time, therefore, it is sufficient to determine the correlation of only spatial coordinates, in the Cartesian coordinate system or, - in a spherical coordinate system. Then the investigated and standard objects are described in the Cartesian coordinate system by the equations in the normal form:

$$\begin{aligned} z &= f(x, y), \\ z_e &= f_e(x, y). \end{aligned}$$

In a spherical coordinate system:

$$\begin{aligned} r &= \varphi(\Theta, \lambda) \\ r_e &= \varphi_e(\Theta, \lambda) \end{aligned}$$

As a rule, objects describing closed figures in the corresponding space are described, or they specify certain finite surfaces, although they can also describe infinite surfaces that are investigated on a finite spatial range.

One of the least complicated figures in the spherical coordinate system is the sphere with the center at the origin point, which is described by the known equation

$$r(\lambda, \Theta) = R.$$

The sphere in the Cartesian coordinates is described by a more complex expression:

$$x^2 + y^2 + z^2 = R^2,$$

where R is the radius of the sphere in the polar and Cartesian coordinate systems.

It is obvious that manipulation with such figure, including the calculation of correlation estimates, is easier to carry out in a spherical coordinate system.

There is a number of figures that are described simply in a spherical system, for example, almost all rotation functions: ellipsoids, paraboloids, hyperboloids, and others.

In the second case when processes are being investigated, while constructing both processes itself and their correlation functions, it is necessary to take into account the time coordinate, since at different times the process may have different spatial coordinates. Therefore, processes need to be investigated in the spatial-temporal coordinate system. Accordingly, it is convenient to represent such processes in a parametric form:

$$\begin{cases} x(t) = f1(t), \\ y(t) = f2(t), \\ z(t) = f3(t). \end{cases} \quad (1)$$

In a spherical coordinate system, the process given parametrically, respectively, is determined by a system of equations:

$$\begin{cases} \lambda(t) = \varphi1(t), \\ \Theta(t) = \varphi2(t), \\ r(t) = \varphi3(t). \end{cases} \quad (2)$$

Then the spherical coordinates in the parametric form have the following form:

$$\begin{aligned} r &= \sqrt{(f_1(t))^2 + (f_2(t))^2 + (f_3(t))^2} \\ \Theta &= \operatorname{arccctg} \frac{f_3(t)}{\sqrt{f_1(t)^2 + f_2(t)^2}} \\ \varphi &= \operatorname{arccctg} \left(\frac{f_2(t)}{f_1(t)} \right) \end{aligned} \quad (3)$$

From (1), (2) and (3) it can be seen that the complexity of expressions will depend on the functions.

We consider a function that describes the sphere in spherical coordinates. This means that, depending on the moment of time, the value of the function will correspond to a certain point on the sphere. This function describes processes in astronomy, in the description of orbits of planets, satellites and other cosmic bodies, the position of objects in radar, guidance systems and a number of other cases. In this case, the expressions will have the simplest form when choosing a coordinate system with the beginning in the scope of the sphere. Such function is given in the polar coordinate system by the equation, where is the radius of the sphere. (fig. 1). When transitioning to the Cartesian coordinate system we use the well-known expressions:

$$\begin{aligned} x &= R \sin \Theta \cos \lambda, \\ y &= R \sin \Theta \sin \lambda, \\ z &= R \cos \Theta. \end{aligned} \quad (4)$$

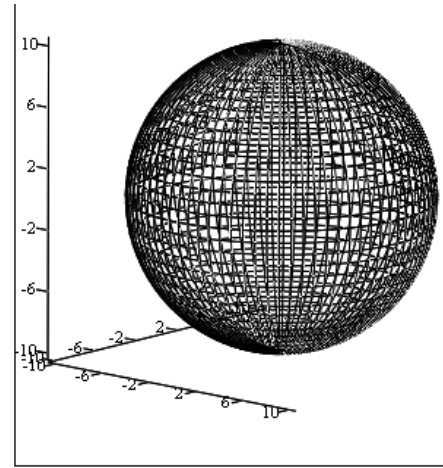


Fig. 1. Graph of the function $r = R$ given in the polar coordinate system.

Another example of a function that describes a number of processes is: $r = R$, $\Theta = \alpha$, where R – radius, α – coordinate angle of latitude.

This function describes the circle (fig. 2) and in practice, it can determine the satellite's orbit, which moves over a certain parallel of the Earth. If the center of the Earth is taken as the coordinates' center, then R - determines the height of the orbit above the surface of the Earth, and the angle of coordinate of latitude is the parallel along which the satellite will move. If, $\alpha = 0$, then the orbit will pass over the equator. If $\Theta = \alpha$ while $0 < \alpha < \pi/2$ the orbit is passing over a certain parallel in the North hemisphere. If $\Theta = \alpha$ while $-\pi/2 < \alpha < 0$ the orbit is passing over a certain parallel in the South hemisphere. If $\alpha = \pi/2$ or $\alpha = -\pi/2$, then the satellite will "hang" over the north or south pole respectively (fig. 2-3).

If the process is periodic with a period T , then you can determine the periodicity of the coordinates λ and Θ , having their own periods of repetition: $\lambda - 2\pi$ - from 0 to 2π ; Θ - also 2π , from $-\pi$ to π . Consequently, the coordinate period λ and Θ finally will be determined from the functions $\varphi1(t)$ for the coordinate λ and the function $\varphi2(t)$ - for the coordinate Θ . It is obvious that when changing the coordinates of time t on the segment $[0, T1]$, $[T1, T2]$ etc., respectively, coordinate λ and Θ should change for the period $[0, 2\pi]$, $[2\pi, 4\pi]$, and $[-\pi, \pi]$, $[\pi, 3\pi]$, etc.

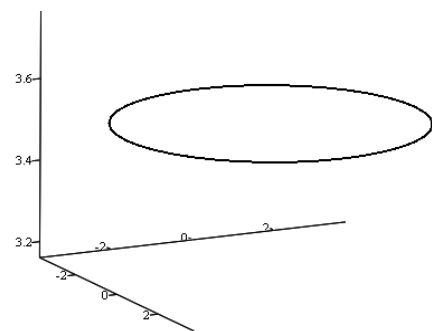


Fig. 2. Schedule of function $r = R$, $\Theta = \alpha$

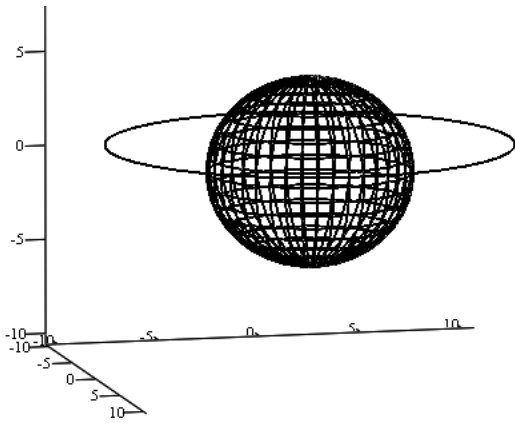


Fig. 3. Graph of an example of a satellite's motion in orbit over a given parallel of the Earth.

If it is difficult or impossible to describe the reference objects or the reference objects functionally, then they are given by an array of coordinate triples, x, y, z – in the Cartesian coordinate system or r, Θ, λ – in the spherical coordinate system of the discrete points of the figures and approximated by certain methods.

III. THEORETICAL FOUNDATIONS OF CONSTRUCTING CORRELATION MODELS IN A SPHERICAL COORDINATE SYSTEM

Having considered the peculiarities of constructing functions in a spherical coordinate system, we turn to the study of the construction of correlation models in a spherical coordinate system and the representation of their graphs in this coordinate system.

Consequently, for the construction of a correlation model of discrete spatial figures [6], the expression for the Cartesian coordinate system will have the form:

$$K_{xe}(p, q) = \frac{1}{n} \cdot \frac{1}{m} \cdot \sum_{i=0}^{N-P-1} \sum_{j=0}^{N-Q-1} f(x_i, y_j) \cdot fe(x_{i+p}, y_{j+q}) \quad (5)$$

where i, j – coordinate indices x, y working sample points;

n, m – the volumes of working samples according to coordinates x, y ;

p, q – indices of displacement by the corresponding coordinates, x, y ;

P, Q – the maximum value of the displacement by the corresponding coordinates x, y ;

N – the total number of discrete points of an object.

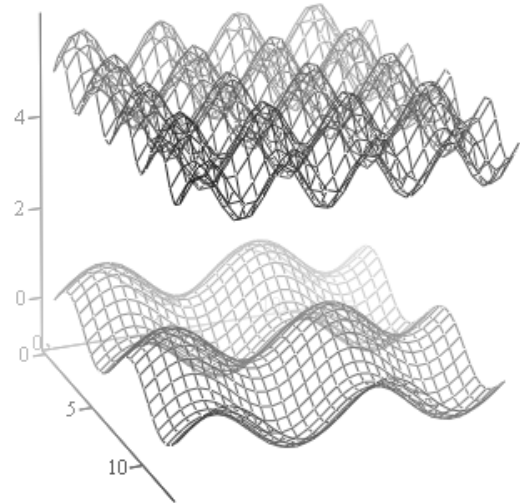
We construct graphs and the corresponding function of correlation (fig. 4), according to (5) for the functions:

$$z_1(x, y) = \frac{\sin x + \sin y}{2} \quad \text{and}$$

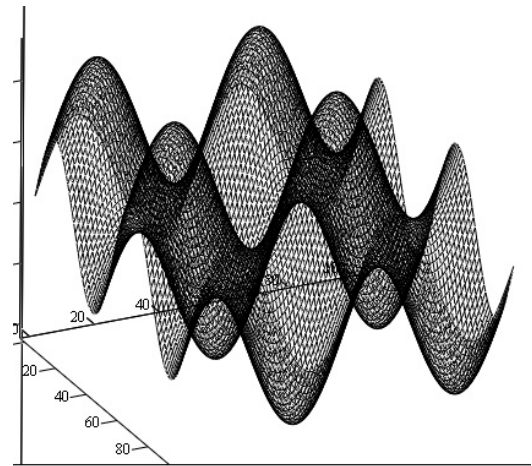
$$z_2(x, y) = 5 + \frac{\sin 2x + \sin 2y}{2}.$$

A similar expression for determining the correlation is obtained for a spherical coordinate system:

$$K_{Rre}(p, q) = \frac{1}{m} \cdot \frac{1}{n} \cdot \sum_{i=0}^{N-P-1} \sum_{j=0}^{N-Q-1} \varphi(\lambda_i, \Theta_j) \cdot \varphi_e(\lambda_{i+p}, \Theta_{j+q}) \quad (6).$$



a)



b)

Fig. 4. a) - Charts functions that correlate $z_1(x, y) = \frac{\sin x + \sin y}{2}$ and $z_2(x, y) = 5 + \frac{\sin 2x + \sin 2y}{2}$;

b) - Charts their covariance function.

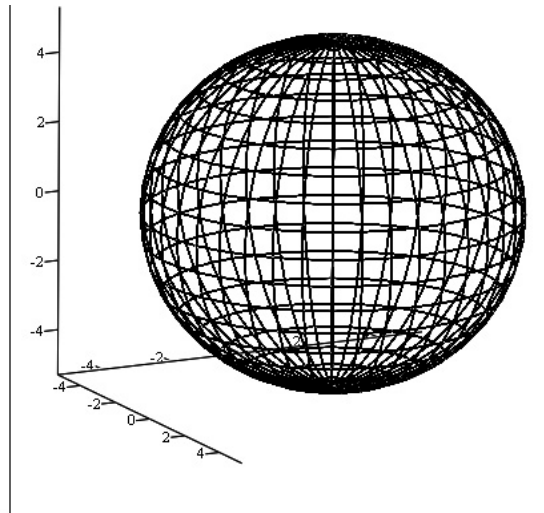
To represent correlation models in a spherical coordinate system we use formulas (3) to translate Cartesian coordinates into spherical ones.

Most often, correlation models are built for time processes. That is $F(x, y, z)$, the function is not a spatial coordinate x, y, z , but a time-dependent t function $f(t)$. In this case, processes in Cartesian coordinates, are set, as a rule, parametrically (1).

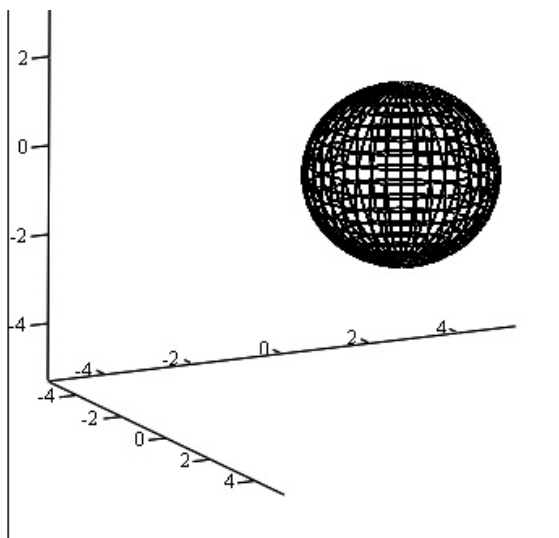
Then spatial coordinates are determined at each time point. In this perspective, it is expedient to consider the time-spatial coordinate system. As already noted, there is a certain category of processes that are much more convenient to represent in a spherical coordinate system. In addition, the analysis of such processes, including correlation, is also

simpler and more convenient in this coordinate system, since mathematical expressions are less complex.

Consider a function that describes the sphere in spherical coordinates. This means that, depending on the moment of time, the value of the function will correspond to a certain point on the sphere. This function describes the processes in astronomy, in the description of orbits of planets, satellites and other cosmic bodies, the position of objects in radar, guidance systems and in a number of other cases. In this case, the expressions will have the simplest form when choosing a coordinate system with the beginning in the center of the sphere. Such functions are given in the polar coordinate system by the equation $r = R = const$, where R – is the radius of the sphere (Fig. 5).



(x1,y1,z1)



(x2,y2,z2)

Fig. 5. Charts of functions of the sphere with radii $R1 = 5$ and $R2 = 2$.

Using the expression of the covariance function (6), for which it is not necessary to center the values of the random process, unlike the correlation function, we will construct a covariance model, the graph of which is presented in fig. 6.

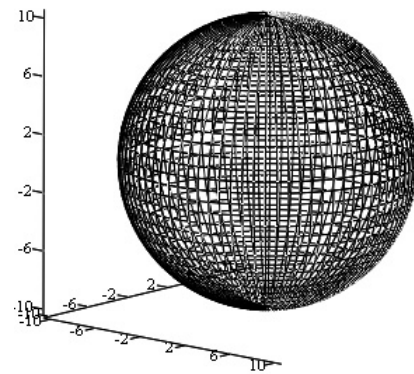


Fig. 6. Representation of a covariance function in a polar coordinate system.

As expected, the covariance function looks like a sphere with a radius $R = R1 \cdot R2 = 2 \cdot 5 = 10$.

Correlation functions in the polar coordinate system require further research and are an effective research tool.

VI. CONCLUSION

The method of constructing correlation models in a spherical coordinate system proposed in the article is a very useful tool for the investigation of two-dimensional stochastic and deterministic signals in radar, astronomy, radio engineering, and other spheres. A number of phenomena and processes in a spherical coordinate system are described by simpler mathematical equations, which simplifies their further analysis. The results of the correlation analysis reflected in the spherical coordinate system are often more visible and understandable.

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