A Mathematical Model of Microsurface Normal Distribution for Specular Bidirectional Reflectance Distribution Function

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Abstract: **This paper presents a new mathematical model of microsurface normal distribution. The proposed model retains a long highlight tail even with roughness coefficient extremely close to zero which matches the behavior of measured material samples. The proposed model obeys the energy conservation law and can be used in Physically-Based Rendering (PBR) shaders to create realistic and visually pleasing specular highlights. The results of using the proposed model are demonstrated and discussed. The highlight shape created by this model is compared against several existing models as well as empirical data.**

Keywords: **Physically-Based Rendering, Specular Reflection, Normal Distribution, Realistic Rendering.**

I. Introduction

Nowadays there are several models of microsurface normal distribution available for specular Bidirectional Reflectance Distribution Function (BRDF) ranging from minimalistic ones like Blinn-Phong to complex models like Cook-Torrance [1], Trowbridge-Reitz [2], Backmann [3], and other models. However, such models suffer from the same problem – with low roughness values the tail of the highlight is not nearly long enough even with the most advanced models.

The research presented in this paper is aimed at creating a new model of normal distribution for specular BRDF that would retain realistically long highlight tail with low roughness values.

II. Related Work

There is a relatively large number of research papers devoted to solving similar tasks.

Thus, Dupuy et al. proposed in [4] a reflectance filtering technique for displacement mapped surfaces called Linear Efficient Antialiased Displacement and Reflectance mapping. The authors assert that their method is compatible with animation and deformation, making it general and flexible.

Gartley et al. presented in [5] the framework for a new p-BRDF prediction tool leveraging the radiometric ray-tracing framework of the Digital Imaging and Remote Sensing Image Generation model. The authors state that predictions from the tool have been verified for clean, randomly rough surfaces against a generalized, analytical p-BRDF model.

In [6], Heitz and d'Eon proposed an importance sampling scheme for microfacet-based BSDFs. The authors claim that the performance of this scheme is suitable for both offline and GPU rendering.

Heitz in [7] presented the masking-shadowing functions (geometric attenuation factors) in microfacet-based BRDFs and provided explanations on their applications.

Hanika et al. in [8] proposed an advancement of microfacet theory based on the Smith model in order to include microsurface multiple scattering at rough material interfaces for reflectance and transmission. The authors compared the predictions made by their model with results obtained by simulating multiple scattering on explicit microsurfaces generated with a noise primitive.

Walteret et al. in [9] provided a review of the microfacet theory and demonstrated how it can be extended to simulate transmission through rough surfaces such as etched glass.

In [10], Dupuy et al. introduce the Symmetric GGX distribution to represent spatially-varying properties of anisotropic microflake participating media.

Despite of availability of these and other promising results a need of new models development still exists.

III. Model Description

The proposed model is based on an assumption that the surface contains cavities of different depths with deeper cavities being less probable. In order to calculate the brightness of a given surface the following steps are performed:

- Calculation of the depth of a cavity that would reflect given light direction vector to the direction of given view vector;
- Calculation of the probability of required cavity.

The reflection of light vector which hits a cavity is demonstrated in Fig. 1.

The height of a cavity which reflects a given light vector into view vector is calculated using following the formula:

$$h=\sqrt{\frac{1}{b^2}-1} \tag{1}$$

where h is the height of the cavity and b is the Blinn term.

The plot of cavity height function in relation to Blinn term is presented in Fig. 2.

After calculation of the cavity height, the probability distribution must be applied. Because Gaussian Distribution

Function does not provide the desired result, the proposed model uses a custom distribution function:

$$f(h,r)=\frac{r}{r+h} \tag{2}$$

where f is the distribution coefficient, h is the height of the cavity, and r is the surface roughness coefficient. This function is not normalized (does not integrate to 1) and, thus, should be divided by its integral to be used as a probability distribution.

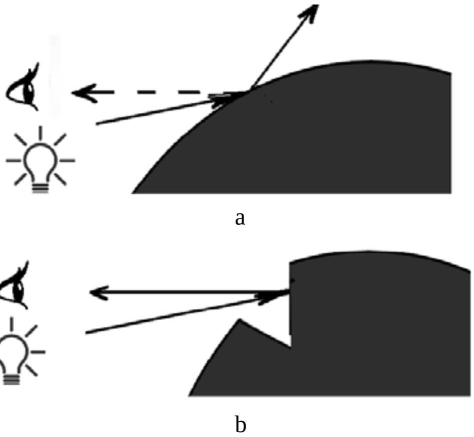

Fig. 1. Reflection cases: a – outside a cavity, b – inside a cavity

However, the integral of this function over $h \in (0; \infty)$ does not converge. But instead it is possible to integrate the total amount of reflected light over $\phi \in (0; \pi)$ where ϕ is the angle between the cavity normal and the surface normal, thus, preserving the energy conservation law.

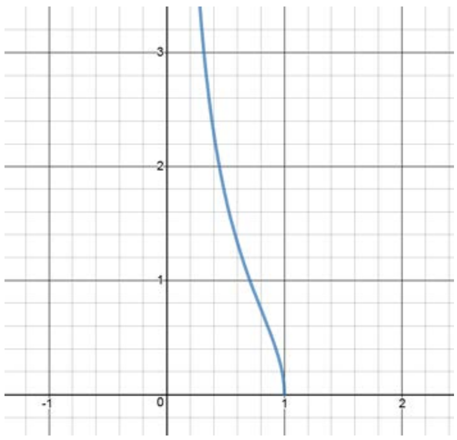

Fig. 2. The plot of cavity height function (vertical axis is the height of theoretical cavity, horizontal axis is Blinn term)

The resulting Normal Distribution Function (NDF) without normalization is:

$$g(\phi,r)=\frac{r}{r+\sqrt{\frac{1}{\cos(\phi)^2}-1}} \tag{3}$$

where f is the distribution coefficient, r is surface roughness, and ϕ is the angle between the surface normal and the cavity normal as well as $\cos(\phi)$ is the Blinn term.

The integral of this function over $\phi \in (0; \pi)$ does converge and is equal to:

$$\int_0^{\pi}\frac{r}{r+\sqrt{\frac{1}{\cos(\phi)^2}-1}}=r\frac{\pi r-2\log(r)}{2r^2+2} \tag{4}$$

The dividing of the proposed distribution function by this integral yields the normalized NDF ready to be used for modelling specular reflections:

$$P(\phi,r)=\frac{2(r^2+1)}{\left(r+\sqrt{\frac{1}{\cos(\phi)^2}-1}\right)(\pi r-2\log(r))} \tag{5}$$

where P (ϕ, r) is the probability of the required cavity, r is the surface roughness coefficient, and $\cos(\phi)$ is the Blinn term.

In order to maintain consistency, it is proposed to use the described NDF with a custom geometric masking function based on the same microsurface model.

The proposed geometric masking function describes the decrease of visible highlight due to microsurface cavity being obscured from light source and from the viewer by adjacent microsurface apex as demonstrated in Fig. 3.

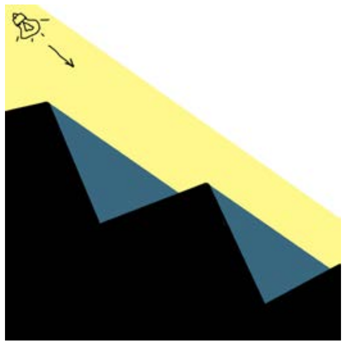

Fig. 3. Demonstration of geometrical masking: surface is black, yellow and blue colors represent light and shadow respectively

The masking term is derived by dividing the length of visible (for view masking) or lit (for light masking) part of microsurface cavity slope by its total length (Fig. 4).

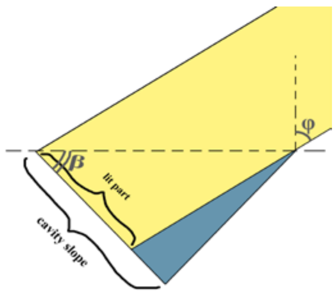

Fig. 4. Masking term derivation (ϕ is the angle between macrosurface normal and light vector, β is the angle between macrosurface and microsurface slope)

The total microsurface slope length can be described as:

$$l_{total} = \frac{1}{2\cos(\phi)} \tag{6}$$

where l_{total} is the total microsurface slope length and ϕ is the angle between the macrosurface normal and the light vector.

The visible part of the slope is equal to:

$$l_{visible} = min\left(\frac{1}{\sin\left(\frac{\pi}{2}+\phi-\beta\right)}, l_{total}\right) \tag{7}$$

where $l_{visible}$ is the visible part of the microsurface slope, l_{total} is the total microsurface slope length, ϕ is the angle between the macrosurface normal and the light vector, and β is the angle between the microsurface slope and the macrosurface.

Thus, the masking coefficient is equal to:

$$m = min\left(\frac{2\cos(\phi)}{\cos(\phi-\beta)}, 1\right) \tag{8}$$

where m is the masking coefficient, ϕ is the angle between the macrosurface normal and the light vector, and β is the angle between the microsurface slope and the macrosurface.

The masking function can be also presented as follows:

$$\frac{2\cos(\phi)}{\cos(\phi-\beta)} = \frac{2\cos(\phi)}{\cos(\phi)\cos(\beta)+\sqrt{(1-\cos(\phi)^2)(1-\cos(\beta)^2)}} \tag{9}$$

Thus, the final masking coefficient formula is:

$$m = min\left(\frac{2\cos(\phi)}{\cos(\phi)\cos(\beta)+\sqrt{(1-\cos(\phi)^2)(1-\cos(\beta)^2)}}, 1\right) \tag{10}$$

where m is the masking coefficient, ϕ is the angle between the macrosurface normal and the light vector, and β is the angle between the microsurface slope and the macrosurface.

The total masking coefficient should be calculated as the lowest value between light masking and view masking:

$$m = min\left(1, min\left(\frac{2\cos(\gamma)}{\cos(\gamma)\cos(\beta)+\sqrt{(1-\cos(\gamma)^2)(1-\cos(\beta)^2)}}, \frac{2\cos(\phi)}{\cos(\phi)\cos(\beta)+\sqrt{(1-\cos(\phi)^2)(1-\cos(\beta)^2)}}\right)\right) \tag{11}$$

where m is the total masking coefficient, γ is the angle between the macrosurface normal and the view vector, ϕ is the angle between the macrosurface normal and the light vector, and β is the angle between the microsurface slope and the macrosurface.

IV. Results and Discussion

The proposed model creates a long highlight tail which looks more similar to empiric measurements as demonstrated in Fig. 5 and Fig. 6.

In fact, the highlight tail of the proposed model is brighter than empirical data and, thus, may require further adjustment to achieve better realism in renders.

Using proposed model for rendering specular highlights creates a smoother and more realistic result as shown in Fig. 7.

a b c d

Fig. 5. Demonstration of highlights: a – measured data from chrome sample, b – proposed model, c – Trowbridge-Reitz model, d – Beckmann model [3]

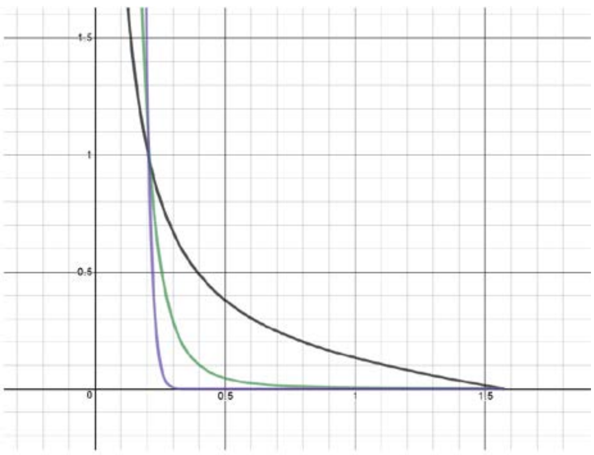

Fig. 6. Comparison of three normal distribution models: the purple line corresponds to Blinn-Phong model, the green line corresponds to Trowbridge-Reitzmodel model, and the black line corresponds to a proposed model

Fig. 7. Demonstration of different normal distribution models: a – Beckmann model, b – Trowbridge-Reits model, c – the proposed model

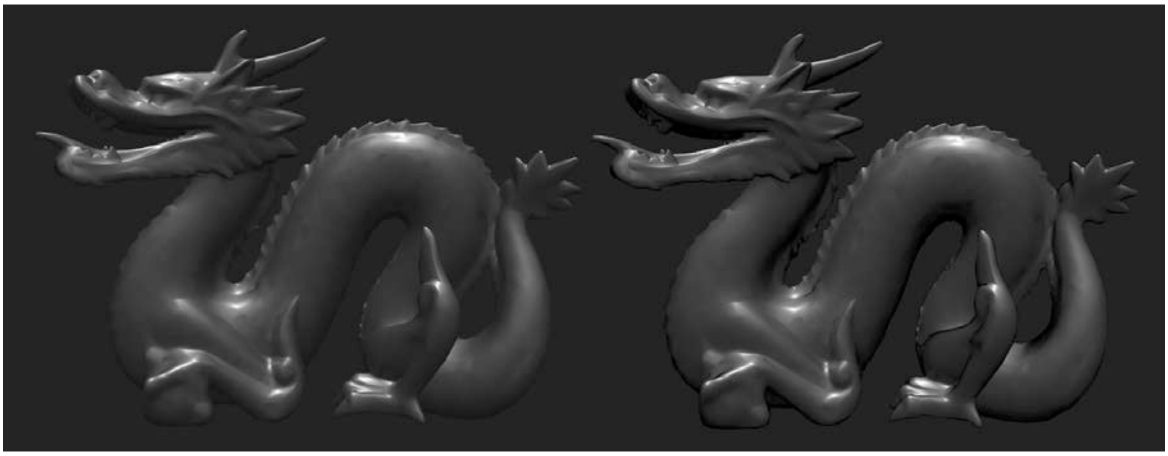

Fig. 8. Proposed masking term demonstration: a – render without a masking term, b – render with the proposed masking term

V. CONCLUSION

The proposed model retains a long highlight tail even with roughness coefficient extremely close to zero yielding more appealing renders. However, it is also computationally intensive. Besides, it must be noted that the highlight tail of the proposed model in fact seems brighter than measured data and, thus, it may need to be adjusted in order to achieve better realism.

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