Inverse Dynamic Models in Chaotic Systems Identification and Control Problems

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Abstract: **Inverse dynamic models approach for chaotic system synchronization in the presence of uncertain parameters is considered. The problem is identifying and compensating unknown state-dependent parametric disturbance describing an unmodelled dynamics that generates chaotic motion. Based on the method of inverse model control, disturbance observers and compensators are synthesized. A control law is proposed that ensures the stabilization of chaotic system movement along master reference trajectory. The results of computational simulation of controlled Rösller attractor synchronization are also presented.**

Keywords: **chaotic system, synchronization, disturbance, identification, inverse model, unknown-input observer.**

I. Introduction

Controlled systems and processes with chaotic dynamics are a matter of unflagging interest in modern control theory and practice [1, 2]. The problem of synchronization of chaotic systems is intensively studied; in this case, control law is designed in such a way that the controlled variables of the slave system follow the reference output of the master system or nonlinear oscillating system stabilized along given reference trajectory in the presence of uncertainties and external disturbances [3, 4].

A typical model of a chaotic system is a linear system with additional nonlinear components dependent on the state, the presence of which determines the appearance of chaotic regimes [5]. Because the system nonlinearity may be treated as a parametric disturbance of nominal model, chaos synchronization problem may be reduced to the disturbance rejection problem, namely, unknown and unmeasurable disturbances eliminating from the systems output along with reference signal tracking.

Recently a number of model-based control methods have been developed for disturbance rejection taking into account the requirements of accuracy, dynamic performance, stability and robustness [6, 7]. In this paper the *inverse model control* approach [8] is applied for chaotic systems synchronization. Inverse models are used for both parametric disturbance identification and compensation, which made it possible to synthesize disturbance decoupling controller, ensure reference signal tracking.

The proposed approach was studied through computational modeling using the example of a controlled Rösller attractor with signal and parametric disturbances.

II. Problem Statement

Consider a state-space model of controlled chaotic system with a distinguished nonlinear component, which causes the emergence of chaotic dynamics and interpreted as an uncertain parametric disturbance

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Nf(x(t),\delta), \\ y_c(t) &= Cx(t),\ \ y_m(t) = Mx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathrm{R}^n$ – chaotic system state vector, $u(t) \in \mathrm{R}^m$ – control variables vector, $f(x(t),\delta) \in \mathrm{R}^q$ – state-dependent parametric disturbance with uncertain parameters δ, $y_c(t) \in \mathrm{R}^r$, $y_m(t) \in \mathrm{R}^p$ – output controlled and measured variables respectively.

Disturbance $f(x(t),\delta) \in \mathrm{R}^q$ may be treated as unknown input signal for system (1).

Matrices $S_{CB}(\alpha_1) = CA^{\alpha_1 - 1}B$, $S_{MN}(\alpha_2) = MA^{\alpha_2 - 1}N$ are known as Markov parameters of system (1).

Without loss of generality, for simplicity reason, we will assume that $\mathrm{rank} S_{CB} = m$, $\mathrm{rank} S_{MN} = m$, where $S_{CB} = S_{CB}(1)$, $S_{MN} = S_{MN}(1)$.

Consider two main inverse model problems:

- *Chaotic system identification*, namely, obtaining unknown parametric disturbance estimate $\hat{f}(t)$ using available measurements $y_m(t)$ and known control signal $u(t)$;
- *Chaotic system control*, namely, control law $u(y(t), y^*(t), \hat{f}(t))$ design, which ensure control goal achieving

$$\overline{\lim} \, || e_c(t) ||^2 \le \varepsilon^*, \quad t \to \infty. \tag{2}$$

where $e_c(t) = y^*(t) - y_c(t)$ – control error, $y^*(t)$ – set-point signal given by the reference model

$$\dot{y}^*(t) = A^* \cdot y^*(t) + y_{ref}(t), \tag{3}$$

ε^* – some sufficiently small constant.

In the chaos synchronization problem, reference model (3) can be considered as a master system [3].

Dynamic system with state vector $\bar{x}(t) \in \mathrm{R}^{n-q}$

$$\dot{\bar{x}}(t) = A^I \bar{x}(t) + B^I u(t) + B_1^I y(t) + B_2^I \dot{y}(t),$$
$$\hat{f}(t) = C^I \bar{x}_k + D^I u(t) + D_1^I y(t) + D_2^I \dot{y}(t), \quad (4)$$

will be referred to as *inverse dynamic model* of system (1), if the following conditions take place: $\left\| \bar{x}(t) - Rx(t) \right\|^2 \to 0$, $\left\| \hat{f}(t) - f(t) \right\|^2 \to 0$, if $t \to \infty$, where $R_{n-q \times n}$ – some aggregate matrix.

Then $\hat{f}(t)$ may be treated as unknown input signal $f(t)$ dynamic estimate, obtained by inverse model (2).

III. Inverse Dynamic Model Design

Let $z(t) = Rx(t) \in \mathrm{R}^{n-p}$ be aggregated auxiliary variables, where R is some aggregate matrix, so that $\mathrm{rank}\left(M^{\mathrm{T}} \; \vdots \; R^{\mathrm{T}} \right) = n$.

Take state vector estimate in the form

$$\hat{x}(t) = P \cdot y_m(t) + Q \cdot \bar{x}(t), \quad (5)$$

where matrices $P \in \mathrm{R}^{n \times p}, Q \in \mathrm{R}^{n \times n-p}$ are such that

$$MP = I_p, \; RQ = I_{n-p}, \; PM + QR = I_n,$$
$$MQ = 0_{p,n-p}, \; RP = 0_{n-p,p}. \quad (6)$$

We obtain the aggregated vector $z(t)$ estimate $\bar{x}(t)$ by minimal-order unknown-input observer (UIO) [9]:

$$\dot{\bar{x}}(t) = \bar{F}\bar{x}(t) + \bar{G}_1 y_m(t) + H\dot{y}_m(t) + \bar{G}_0 u(t). \quad (7)$$

The UIO (7) parameters are determined from disturbance estimate invariance conditions [9, 10]

$$\left(R - \bar{H}M \right) A - \bar{F}\left(R - \bar{H}M \right) = \bar{G}M,$$
$$RN - \bar{H}MN = 0, \; \bar{G}_0 - RB = 0, \; \bar{G}_1 = \bar{G} - \bar{F}\bar{H}. \quad (8)$$

A solution of linear matrix equations (8) are obtained as

$$\bar{F} = R\Pi_N AQ, \;\; \bar{G}_0 = RB,$$
$$\bar{G}_1 = R\Pi_N AP, \; \bar{H} = RNS_{MN}^+, \quad (9)$$
$$\Pi_N = I_n - BS_{MN}^+ M,$$

Taking the unknown disturbance estimate as

$$\hat{f}(t) = N^+ \left(\dot{\hat{x}}(t) - A\hat{x}(t) - Bu(t) \right), \quad (10)$$

we can obtained the minimal-order state and disturbance observer in the form of system (1) inverse model [10]:

$$\dot{\bar{x}}(t) = R\Pi_N AQ \cdot \bar{x}(t) + R\Pi_N AP \cdot y_m(t) +$$
$$+ RNS_{MN}^+ \cdot \dot{y}_m(t) + R\Pi_N B \cdot u(t),$$
$$\hat{x}(t) = P \cdot y_m(t) + Q \cdot \bar{x}(t), \quad (11)$$
$$\hat{f}(t) = \bar{C}_N [\dot{y}_m(t) - MAQ \cdot \bar{x}(t) -$$
$$- MAP \cdot y_m(t) - S_{MB} u(t)],$$

where $\Pi_N = I_n - NS_{MN}^+ M$, $\Omega_N = I_p - S_{MN} S_{MN}^+$, $C_N = S_{MN}^+ + N^+ P \Omega_N$.

From (1), (11) it follows, that estimate errors vectors $e_x(t) = x(t) - \hat{x}(t), \;\; e_f(t) = f(x(t),t) - \hat{f}(t)$ are given by the equations:

$$\dot{\bar{e}}_x(t) = \bar{F}\left(R\right) \cdot \bar{e}_x(t),$$
$$e_x(t) = Q \cdot \bar{e}_x(t) \quad (12)$$
$$e_f(t) = -C_N MAQ \cdot \bar{e}_x(t).$$

III. Inverse Model-Based Controller Design

The disturbance rejection control law will be constructed as a function of reference signal and disturbance estimate:

$$u^*(t) = S_{CB}^{-1} \cdot [y_{ref}(t) + C_A \hat{x}(t) - S_{CN} \hat{f}(t)],$$
$$C_A = A^* C - CA. \quad (13)$$

If system structure non-singularity condition takes place

$$\mathrm{rank}\, \bar{S} = m + q, \quad \bar{S} = \begin{pmatrix} I_m & S_{CB}^{-1} S_{CN} \\ C_N S_{MB} & I_q \end{pmatrix} \quad (14)$$

or equivalently

$$\det \Phi \neq 0, \;\; \Phi = I_q - C_N S_{MB} S_{CB}^{-1} S_{CN}, \quad (15)$$

disturbance estimate may be eliminated from the controller equations, which is therefore be regarded as disturbance decoupling controller.

In reality, situations often arise when conditions (14), (15) are not met. In such a case the realizable control law may be obtained using the disturbance estimates, dynamically transformed by the internal auxiliary "fast" filter with small parameters.

As a result, realizable controller are designed by including in its structure an additional internal low-pass filter with small time constant [11]:

$$u^*(t) = S_{CB}^{-1} \cdot [y^*(t) + C_A \hat{x}(t) - S_{CN} \tilde{f}(t)],$$
$$\varepsilon \dot{\tilde{f}}(t) = -\tilde{f}(t) + (1 - \mu) \cdot \hat{f}(t), \quad (16)$$

where $0 < \varepsilon << 1, \quad 0 < \mu << 1$ - small filter parameters.

From (13), (16) follows, that disturbance compensator equation with internal additional filter take the form:

$$\begin{aligned} &\varepsilon\dot{\tilde{u}}(t) = -\mu\tilde{u}(t) + (1-\mu)\cdot[\varphi_1(t) + \\ &\qquad + S_{CB}^{-1}S_{CN}\varphi_2(t)], \\ &u^*(t) = \tilde{u}(t) + \varphi_1(t), \\ &\varphi_1(t) = S_{CB}^{-1}\cdot[y_{ref}(t) + C_A\hat{x}(t)], \\ &\varphi_2(t) = C_N\cdot[\dot{y}_m(t) - MAQ\cdot\bar{x}(t) - MAP\cdot y_m(t)]. \end{aligned} \tag{17}$$

III. Chaotic System Inverse Model Control

As an example of proposed approach consider inverse model control of the Rösller attractor under uncertainties:

$$\begin{aligned} &\dot{x}_1(t) = -x_2(t) - x_3(t), \\ &\dot{x}_2(t) = x_1(t) + ax_2(t) + u_1(t) + f_1(t), \\ &\dot{x}_3(t) = -cx_3(t) + u_2(t) + f_1(t) + f_2(x_1(t), x_3(t)), \end{aligned} \tag{18}$$

where
$f_1(t) = \delta_f$, $f_2(x_1(t), x_3(t)) = \delta_c x_3(t) + (1+\delta_x)x_1(t)x_3(t)$, are input and parametric disturbances respectively with $\delta_f, \delta_c, \delta_x$ uncertain parameters.

Using the measurements $y_1(t) = x_1(t), \quad y_2(t) = x_3(t)$ find the control so the controlled output $y_c(t) = x_1(t)$ will track set-point signal $y^*(t)$, generated by reference model

$$\ddot{y}^*(t) + \alpha_1\dot{y}^*(t) + \alpha_0 y^*(t) = y_{ref}(t). \tag{19}$$

The control law, which ensures attractor synchronization with reference model, is the following:

$$\begin{aligned} &u_2(t) = (\alpha_0 - 1)\cdot\hat{x}_1(t) + (k - a - \alpha_1)\cdot\hat{x}_2(t) + \\ &\qquad + (c - \alpha_1)\cdot\hat{x}_3(t) - 2\hat{f}_1(t) - \tilde{f}_2(t) - y_{ref}(t), \\ &u_1(t) = -k\hat{x}_2(t), \\ &\varepsilon\dot{\tilde{f}}(t) = -\tilde{f}(t) + (1-\mu)\cdot\hat{f}_2(t), \end{aligned} \tag{20}$$

The state estimates for system (18), obtained by reduced-order UIO, are:

$$\begin{aligned} &\dot{\bar{x}}_1(t) = \rho_1\bar{x}_1(t) + \bar{x}_2(t) + \\ &\qquad + (1 + \pi_1\rho_1 + \pi_2)\cdot y_1(t) + \pi_1 y_2(t), \\ &\dot{\bar{x}}_2(t) = \pi_2\bar{x}_1(t) + \pi_1\pi_2 y_1(t) + \pi_2 y_2(t), \\ &\hat{x}_1(t) = y_1(t), \\ &\hat{x}_1(t) = \bar{x}_1(t) + \pi_1 y_1(t), \ \hat{x}_3(t) = y_2(t), \end{aligned} \tag{21}$$

where $\rho_1 = (\pi_1 + a - k)$, π_1 π_2 are tuning parameters.

Corresponding disturbance estimates are

$$\begin{aligned} &\hat{f}_1(t) = \bar{x}_2(t) + \pi_2 y_1(t), \\ &\hat{f}_2(t) = \dot{y}_2(t) + cy_2(t) - \bar{x}_2(t) - \pi_2 y_1(t) - u_2(t). \end{aligned} \tag{22}$$

As a result disturbance decoupling controller with internal filter equation is obtained in the form:

$$\begin{aligned} &\varepsilon\dot{\bar{u}}(t) = \nu_1\bar{x}_1(t) - \bar{x}_2(t) + (\zeta_1 - \pi_2)y_1(t) - \alpha_1 y_2(t), \\ &u_2(t) = \bar{u}(t) + \nu_1\bar{x}_1(t) - 2\bar{x}_2(t) + \\ &\qquad + (\zeta_1 - 2\pi_2)\cdot y_1(t) + (c - \alpha_1 - \varepsilon^{-1})\cdot y_2(t), \\ &\zeta_1 = \alpha_1 + \nu_1\pi_1 - 1, \quad \nu_1 = k - \alpha_1 - a \end{aligned} \tag{23}$$

Proposed disturbance observer and decoupling controller are investigated by computational simulation.

Simulation results for Rösller attractor model parameters $a = 0.2, c = -5.7$, observer and controller parameters $\pi_1 = -1, \pi_2 = -2, \varepsilon = 0.01, \mu = 0, k = 2.2$, and reference model parameters $\alpha_0 = 5, \alpha_1 = 6$ are presented below.

Disturbance $f_1(t)$ was modeled input signal disturbance as a step wave function, reference model input signal $y_{ref}(t)$ adopted in the form of harmonic function.

At Fig.1, 2 the state variables and phase plane of controlled Rösller disturbed attractor are presented.

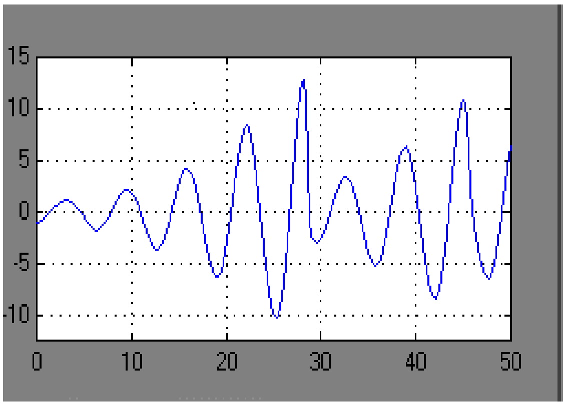

Fig.1. Dynamics of the disturbed attractor.
State variable $y_1(t)$

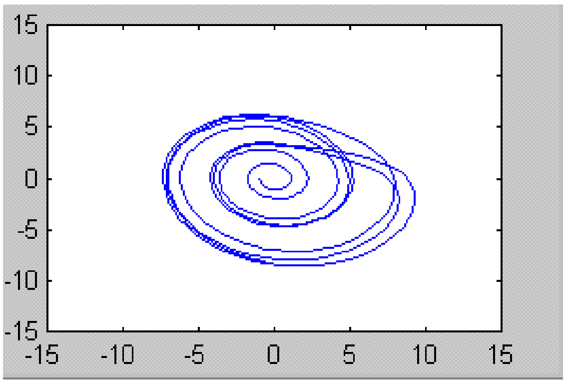

Fig.2. Dynamics of the disturbed attractor.
Phase plane $(y_1(t), y_2(t))$

Disturbances estimations obtained by (21), (22) are depicted in Fig. 3, 4 and control and output variables obtained in accordance the control law (20), (23) are presented at Fig. 5, 6.

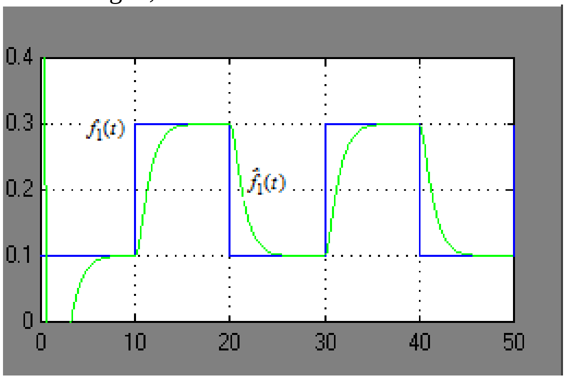

Fig.3. Disturbances estimation $f_1(t)$, $\hat{f}_1(t)$ in open-loop system

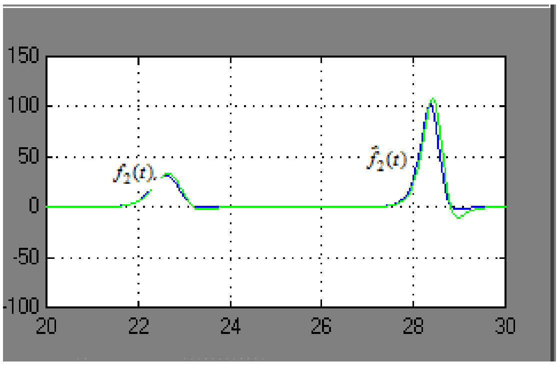

Fig.4. Disturbances estimation $f_2(t)$, $\hat{f}_2(t)$ in open-loop system

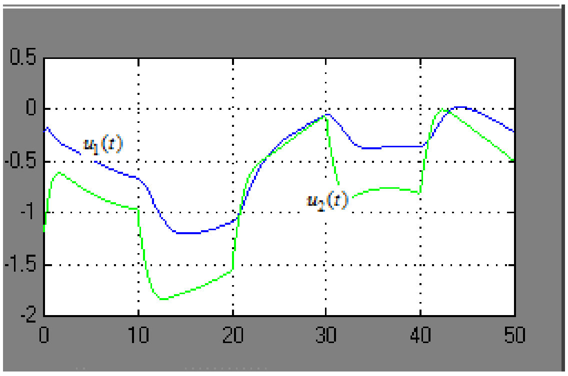

Fig.5. Control variables $u_1(t), u_2(t)$

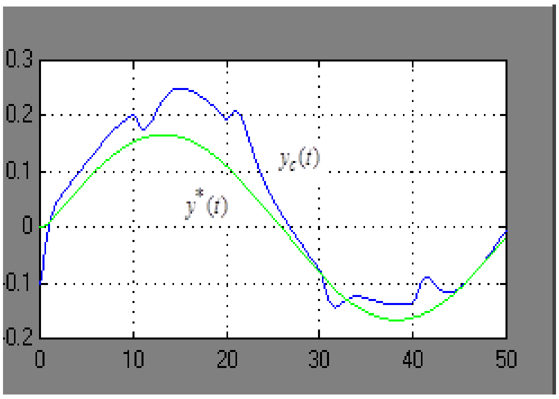

Fig.6. Set-point signal $y^*(t)$ and output variables $y_c(t)$

Simulation results for chaotic system synchronization problem demonstrated high accuracy of disturbances decoupling for broad range of parameters deviation.

IV. Conclusion

In this paper we presented inverse model-based approach to chaotic systems synchronization problem. The proposed method allows us to decompose the problem into the stage of structural synthesis of inverse models and their parametric synthesis or optimization. This significant advantage of the method of inverse dynamic models is the possibility of real-time reconstruction of signals of complex shape in the absence of information on their structure. This, in turn, makes it possible to efficiently solve problems of compensation of nonlinear state-dependent, which makes it possible to suppress sources of chaotic dynamics and simplifies the solution of synchronization problems. Thus proposed approach seems to be quite universal and can be used to solve various problems of controlling chaotic systems.

The implementation of the proposed control requires differentiating the measured output signals in real time, for which differentiators based on sliding modes can be used. Further development of the proposed approach is associated with the development of robust methods for inverse models design under conditions of uncertain deviations of the parameters of the chaotic object model.

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