RESEARCHES OF THE COMBINATORIAL MODELS FOR INNOVATIVE INFORMATION TECHNOLOGIES

Volodymyr Riznyk [1) 2)]

[1)] Lviv Polytechnic National University, 12 S.Bandera Str., 79013, Lviv, Ukraine, rvv@polynet.lviv.ua, http://iknit.lp.edu.ua/riznyk
[2)] University of Technology and Life Sciences Bydgoszcz, 7 Kaliskiego Ale, 85-796, Poland, wriz@utp.edu.pl

Abstract: *The paper presents a new mathematical principle of innovative techniques for improving the quality indices of engineering devices and systems with non-uniform structure (e.g. coding systems) with respect to transmission speed, positioning precision, resolving ability, and functionality, using novel design based on remarkable properties and structural perfection of one- and multidimensional models of the systems, namely the concept of Ideal Ring Bundles (IRB)s prospected from basic laws of the world-wide harmony. Research into the underlying mathematical principle provides an ability to reproduce the maximum number of combinatorial varieties in the systems with a limited number of elements and bonds. This approach make it possible to configure systems with optimal placement of structural elements in spatially or temporally distributed systems, using the appropriate mathematical apparatus of contemporary combinatorial theory.*

Keywords: *Combinatorial model, Information technology, Numerical model, Coding system, Transmission speed, Resolving ability, Multidimensional model, Vector data coded design, Ideal Ring Bundle, World-wide harmony.*

1. INTRODUCTION

Problem of structural optimization in systems engineering relating to finding the best placement both elements and bonds of the system as well as improving its quality indices, is known, to be connected not only with number theory and combinatorial analysis but also with research of the fundamental laws of real world, such as symmetry laws [1], [2] and aurea section [2]. Research into underlying mathematical area involves investigation of novel techniques based on Combinatorial Mathematics, and Combinatorial Sequencing Theory, namely the concept of Ideal Ring Bundles [3], which can be used for finding optimal solutions for some problems in systems engineering and information technologies.

2. NUMERICAL MODEL OF HIGH-PERFORMANCE SYSTEMS

The ordered chain approach to the study of elements and events is known to be of widespread applicability when applied to the problem of finding the optimum ordered (non-redundant) arrangement of structural elements in a distributed technological or computing system. Let us regard an *n*-stage sequence of distinct positive integers $K_n = \{k_1, k_2, \dots k_n\}$ as being cyclic, so that k_n is followed by k_1.

We call this a ring sequence (Fig.1). A sum of consecutive terms in the ring sequence can have any of the *n* terms as its starting point, and can be of any number of terms from 1 to *n*-1. In addition, there is the sum of all *n* terms, which is the same independent of the starting point. Hence, the maximum number of distinct sums S_n of consecutive terms of the ring sequence is given by:

$$S_n = n(n-1) + 1 \tag{1}$$

Here is graphical model of an *n*-stage ring sequence $K_n = \{k_1, k_2, \dots k_n\}$ (Fig.1).
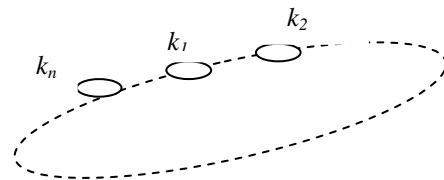

Fig.1 – Graphical model of an *n*-stage ring sequence $K_n = \{k_1, k_2, \dots k_n\}$

If we require all terms in each sum to be consecutive elements of the sequence, for which the set of all S_n circular sums consists of the numbers from 1 to $S_n = n(n-1)+1$ that is each number occurs exactly once is called *Ideal Ring Bundle* (IRB) with

parameters n, S_n. Here is an example of an IRB with parameters n=4, S_n =13, namely {1, 3, 2, 7} (Fig.2).
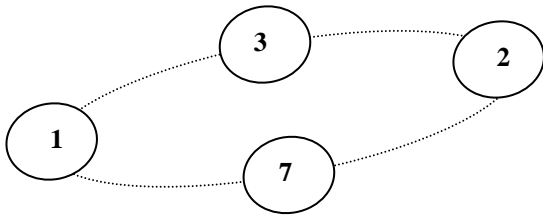

Fig. 2 – The Ideal Ring Bundle with n=4, S_n =13

To see this, we observe complete set of circular sums calculated from the IRB {1,3,2,7} (Table 1).

Table 1. Circular sums of the IRB {1,3,2,7}

1=1	2=2	3=3	4=1+3
5=3+2	6=1+3+2	7=7	8=7+1
9=2+7	10=2+7+1	11=7+1+3	12=3+2+7
13=1+3+2+7			

Note that if we allow summing over more than one complete revolution around the ring, we can obtain all positive integers as such sums. Thus:

14 = 1+3 +2 +7 +1, 15 = 2 +7 +1+3 +2,
16 = 3+2 +7 +1 +3, etc.

One-dimensional (1-D) Ideal Ring Bundle is an ordered numerical construction with n distinct integers, which form perfect partitions of finite interval [1,s] of integers. The sums of connected sub-sequences of 1-D IRB enumerate the set integers [1,s] exactly R-times [3].

The favorable qualities of numerical models provide many opportunities to apply them to advanced information technology and numerous branches of science.

3. IRB CODING SYSTEMS

Concept of the Ideal Ring Bundles can be used for finding optimal solutions for some problems of information computational processes. For example, Ideal Ring Bundles make it possible to configure optimum coding system, based on so-called IRB Monolithic Binary Code. This code forms binary code combinations, which all symbols "1" as well as symbols "0" are arranged together [4]. The IRBs provide an ability to reproduce the maximum number of the "monolithic" code combinations (all symbols "1" as well as symbols "0" are arranged together) in the coding system with a limited number of code word digits. First of all we can see computational processes, based on the IRB Monolithic Binary Code.

Here is an example of one-dimensional Monolithic Binary Code based on the IRB {1,3,2,7} with parameters n=4, S=13:

Table 2. One-dimensional Monolithic Binary Code based on the IRB {7, 2, 3, 1}

Number	Code combination
0	0000
1	0001
2	0101
3	0010
4	0011
5	0110
6	0111
7	1000
8	1001
9	1100
10	1101
11	1011
12	1110
13	1111

Table 2 contains the complete set of binary code combinations for coding the numbers 0, 1,…13.

Underlying combinatorial constructions can be represented as mathematical model of optimum coding system, based on the IRBs conception.

4. GENERALIZED IDEAL RING BUNDLES

Next, we consider a more general type of IRB, where the S_n circular sums of consecutive terms give us each integer value from 1 to N, for some integer N, exactly R times, as well as the value $N - 1$ (the sum of all n terms) exactly once. Here we see that:

$$N = n(n\text{-}1)/R \qquad (2)$$

Here is a graph of the IRB with $n = 4$, $R = 2$ and $N = 6$, where k_1=1, k_2,=1, k_3=2, k_4=3} (Fig.3).

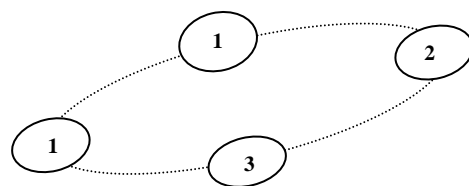**Fig. 3 – Graph of the IRB with n =4, N =6, R =2**

Sums of consecutive terms for the IRB {1, 1, 2, 3} are:

1:	1,	and	1,
2:	1 + 1,	and	2,
3:	1 + 2,	and	3,
4:	1 + 1 + 2	and	3 + 1,
5:	2 + 3	and	3 + 1 + 1,
6:	1 + 2 + 3	and	2 + 3 + 1

Using equation (2) easy to calculate sums of all n terms for double (R=2) IRB, namely: 2, 4, 7, 11, etc. We say this numerical set is generative double-IRBs

row.

5. SEGMENTATION OF IDEAL RING BUNDLES

Let us regard of the IRB{1,3,2,7} depicted above (Fig.2) and the IRB{1,1,2,3} (Fig.3) as graphic vision of an image segmentation of the IRBs (Fig.4).
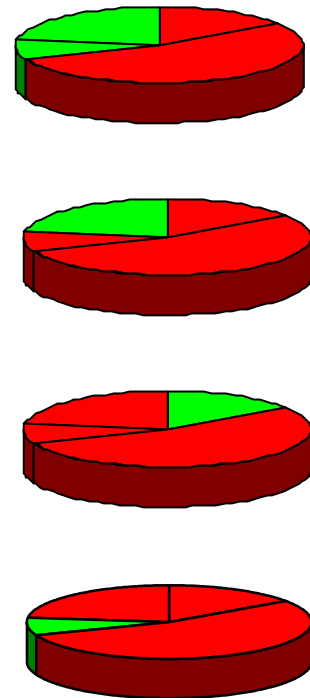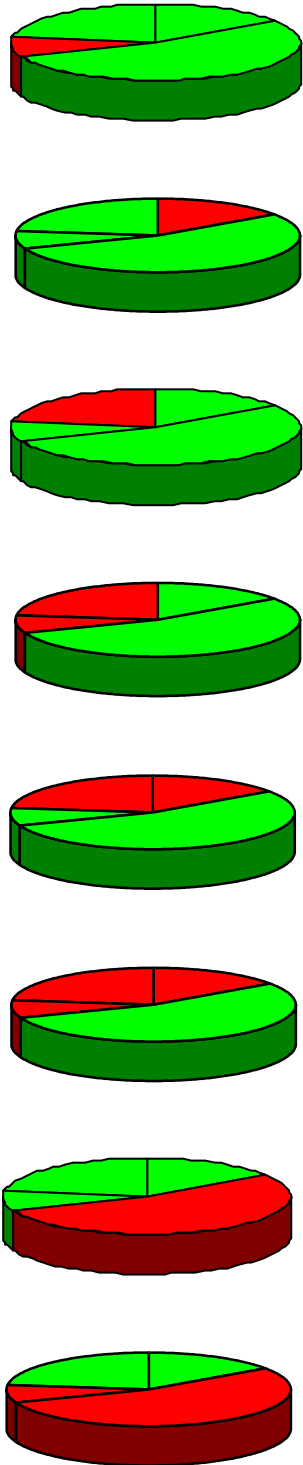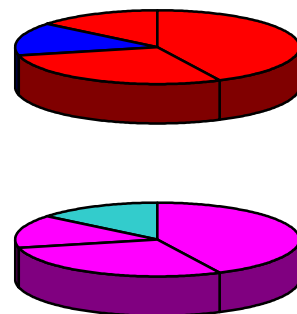

Fig. 4 – Graphic segmentation of the IRB {1,3,2,7}

Regarding picture (Fig.4), easy to see exhaustive set of harmonious two-body relationships (here is red and green) from 1:12 to 1:12 obtained exactly once ($R = 1$) each of them using the IRB {1, 3, 2, 7} image segmentation.

So, 13 is a lucky number, because it allows on the perfect partition 13=1+3+2+7 and provides an ability to reproduce the maximum number of harmonious two-body relationships. The fact makes it possible to develop of new approach in research of the discrete structures from epistemological point of view or "nature and nurture" [2].

Now, let us regard a graphic segmentation of the Ideal Ring Bundle {1, 1, 2, 3}. Here is exhaustive set of harmonious two-body relationships from 1:6 to 6:1 obtained exactly twice ($R = 2$) each of them over the IRB {1, 1, 2, 3} (Fig. 5).
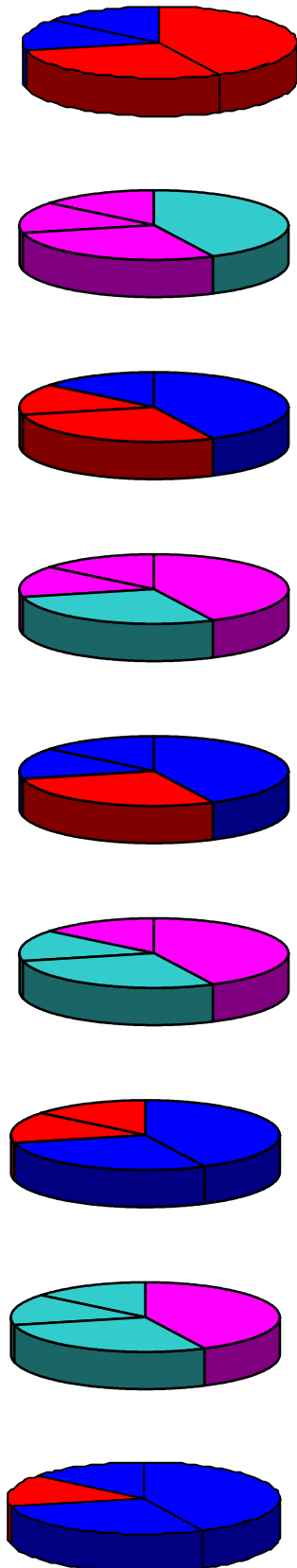
Fig. 5 – Graphic segmentation of the IRB {1, 1, 2, 3}

Observing graphic segmentation of the IRB{1, 1, 2, 3} (Fig.5), we see the exhaustive set of harmonious two-body relationships (here is blue and red, and sky-blue and lilac) from 1:6 to 1:6 obtained exactly twice (*R*=2) each of them. So, 7 is lucky number, because it provides an ability to reproduce the exhaustive set of harmonious two-body relationships.

6. PERFECT NON-SYMMETRY AND SYMMETRY OF THE IDEAL RING BUNDLES

We can see, that neither IRB {1, 3, 2, 7} nor IRB {1, 1, 2, 3} are symmetric structures. Now let us consider a central symmetric figure of order 7 (Fig.3). Easy to see, that it is a chart of the IRB {1, 1, 1, 1, 1, 1, 1} with *n*=7, S_n =7, and *R* =7. This chart consists of two IRBs, which penetrate with each other, namely the IRB {1, 1, 2, 3} (black bubbles) and the IRB {1, 2, 4} (red bubbles).
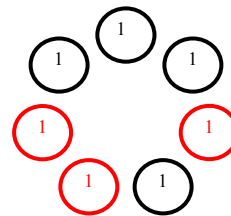

Fig. 6 – Bubble chart of the symmetric IRB {1, 1, 1, 1, 1, 1, 1} joined two non-symmetric IRBs {1, 1, 2, 3} and {1, 2, 4}

There are, it is known, numerous numbers of the underlying perfect triads of IRBs, which form so-called universal numerical information field [5]. It is, essentially, a fundamental property of the space-time. The mutual connection between symmetry and non-symmetry of the IRBs provides a better understanding of physics of space-time, and role of geometric structure in the behaviour of natural and man-made objects.

7. IDEAL RING BUNDLE AS SYMMETRIC OBJECT

Next, we regard the *n*-stage ring sequence $K_{2D}=\{(k_{11},k_{12}),(k_{21},k_{22})...(k_{n1},k_{n2})\}$, where we require

all terms in each circular vector-sum to be consecutive 2-stage sequences as elements of the sequence. A circular vector-sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to n-1. An n-stage ring sequence K_{2D}, for which the set of all circular vector-sum forms two-dimensional grid, where each node of the grid occurs exactly R-times, is named a two-dimensional Ideal Ring Bundle (2-D IRB).

Here is two-dimensional n-stage ring sequence IRB with four (n=4) terms in the ring topology, where k_1= (0,2), k_2= (1,0), k_3=(1,1), k_4=(2,2), which grid chart is depicted below (Fig.7).
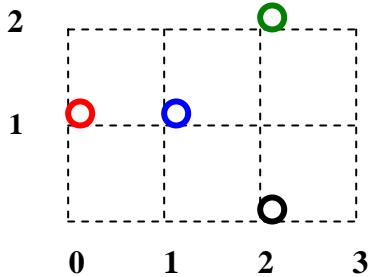

Fig. 7. Grid chart of two-dimensional Ideal Ring Bundle (2-D IRB) of four (n=4) vectors {(0, 2), (1, 0), (1, 1), (2, 2)}

We can calculate easy the all circular two-dimensional vector-sums, taking modulo m_1 =3 for the first component of vector-sum and modulo m_2 =4 for the second its component:

$$(0, 0) \equiv (2, 2) + (0, 2) + (1, 0),$$
$$(0, 1) \equiv (1, 1) + (2, 2) + (0, 2),$$
$$(0, 2) = (0, 2),$$
$$(0, 3) \equiv (1, 1) + (2, 2),$$
$$(1, 0) = (1, 0),$$
$$(1, 1) = (1, 1),$$
$$(1, 2) \equiv (0, 2) + (1, 0),$$
$$(1, 3) \equiv (1, 0) + (1, 1) + (2, 2),$$
$$(2, 0) \equiv (2, 2) + (0, 2),$$
$$(2, 1) \equiv (1, 0) + (1, 1),$$
$$(2, 2) = (2, 2),$$
$$(2, 3) \equiv (0, 2) + (1, 0) + (1, 1)$$

So long as the vectors (0, 2), (1, 0), (1, 1), (2, 2) of the ring sequence themselves are circular vector-sums too, the circular vector-sums set configure the 3×4 matrix as follows:

$$(0, 0) \quad (0, 1) \quad (0, 2) \quad (0, 3)$$
$$(1, 0) \quad (1, 1) \quad (1, 2) \quad (1, 3)$$
$$(2, 0) \quad (2, 1) \quad (2, 2) \quad (2, 3)$$

The result of the calculation forms the 3×4 grid which exhausts the circular 2-D vector-sums and each of its meets exactly once (R=1). So, the ring sequence of the 2-D vectors {(0, 2), (1, 0), (1, 1), (2, 2)} is two-dimensional Ideal Ring Bundle (2-D IRB) with n=4, R=1, and m_1 =3, m_2 =4.

It is easy to see, that 2-D IRB {(0, 2), (1, 0), (1, 1), (2, 2)} has the mirror symmetry, as well as differences of coordinates for all pairs of elements (bubbles on the grid chart) form a set of non-identical positions of 2-D vectors. So, the 2-D IRB is both symmetric and non-redundant structure at the same time. There are lots of symmetric and non-redundant 2-D IRBs. It is known, we can configure numerous numbers of one- and multidimensional IRBs [4].

As we see a whole can be partitioned into two parts using corresponding couple of rays, where one part of two partners of an arbitrary polarity is designed with the sign "+" and the other part of this polarity is designed with the sign "-", so in the following we shall always speak of the IRB polarity.

Perfect Distribution Law (PDL), based on the remarkable properties and structural perfection of one- and multidimensional IRBs are as follows:

A whole (space-time) can be partitioned perfectly, namely the sums of connected parts of the whole enumerate the set of harmony rising its parts exactly R-times with the smallest possible number of intersections.

The main characteristics of the PDL [5]:
1. A whole can be partitioned perfectly by infinitely great number of ways.
2. The minimal share of a whole in perfect partitioning can be anyone small.
3. The perfect partitioning can be useful to objects of anyone vector space dimensionality.

8. PERFECT VECTOR DATA CODING DESIGN

The development of new direction in fundamental and applied research in information technologies is connected with vector data coding design, based on the concept of Ideal Ring Bundles. For example, underlying combinatorial construction can be represented as mathematical model of optimum coding system, based on so-called "Monolithic Binary Code" (MBC). This code forms binary code combinations which all symbols "1" as well as "0" are arranged together. For example, two-dimensional vector data MBC formed on the 2-D IRB {(0,2), (1,0),(1,1),(2,2)}, is given in the Table 3.

Table 3. 2-D MBC based on the irb {(0,2), (1,0), (1,1), (2,2)}

Vector	Code	Vector	Code	Vector	Code
(0,0)	1101	(1,0)	0100	(2,0)	1001
(0,1)	1011	(1,1)	0010	(2,1)	0110
(0,2)	1000	(1,2)	1100	(2,2)	0001
(0,3)	0011	(1,3)	0111	(2,3)	1110

Table 3 contains set of binary code combinations for coding of all 2-D vectors on the 3×4 − matrix from (0,0) (code combination 1101) to (2,3) (code combination 1110), where each of them has been coded in circular 2-D MBC. The remarkable properties of the MBC provide its some advantages over the rest codes, involving simplicity of error detecting and correcting as well as high-speed operation.

8. CONCLUSION

The Perfect Distribution Law (PDL) discovers, essentially, a new scientific conception for development fundamental and applied research in information technologies, based on the idea of "perfect" combinatorial constructions- Ideal Ring Bundles (IRB)s, and the development of new directions in fundamental and applied research in area of information technologies, for improving such quality indices as reliability, precision, speed, resolving ability, and functionality. Structural perfection and remarkable properties of one- and multidimensional IRBs provide an ability to reproduce the maximum number of combinatorial varieties in the systems with a limited number of elements and bonds. It was IRBs provide many opportunities to apply them to advanced information technologies. The innovative methodologies based on combinatorial techniques of the Ideal Ring Bundles theory can be used for finding optimal solutions for wide classes of technological problems. Perspective information technologies and coded design prospected from the PDL are for example high performance vector data coding and self-checking code, vector computing machinery and low side-lobe antenna design [4]. The mutual connection between symmetry and non-symmetry of the IRBs provides a better understanding of physics of space-time, and role of geometric structure in the behaviour of natural and man-made objects. A perfection, beauty and harmony exists not only in the abstract models but in the real world also.

9. REFERENCES

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Volodymyr V. Riznyk born in Polis'ke, Kyiv Region, Ukraine, 1940. Education: Diploma, Power Engineering, 1962; Diploma, Radio Engineering, 1967; PhD Degree, Cybernetics and Theory of Information, Physical and Mechanical Institute, Academy of Sciences, Ukr. SSR, 1980; D Sc, Mathematical Modeling and Mathematical Techniques for Scientific Researches, Vinnytsia State Technical University, 1994. Full professor in Lviv Polytechnic National University, 1995, University of Technology and Life Sciences Bydgoszcz, Poland, 1996.

Scientific interests: researches and applications of combinatorial models and techniques to problems of information technologies.