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ISSN 1727-6209
International Journal of Computing

DEVELOPMENT OF THE GUARANTEED ESTIMATION ROBUST ALGORITHM OF LINEAR CONTROLLED SYSTEM STATES

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Abstract: *Ellipsoidal approximation of the ellipsoid and hyperlayer crossing has been considered as a basis of the algorithm of states estimation of the linear controlled system whose set of possible states is represented with an ellipsoid, and observations – with a hyperlayer. This representation is considered as an analogue of Kalman filter. The conditions of a priori system state and a posteriori measurement information compatibility and sensitivity of the algorithm to a choice of its parameters have been investigated. Dependence of the system state estimate improvement on a relative width of the hyperlayer of a set of observations has been shown. The obtained algorithm in comparison with the known solutions at minor degradation of accuracy is much easier in realization and stabler in operation from the standpoint of prior guesses violation.*

Keywords: *Linear controlled system, a set of attainability, a hyperlayer of a set of system states observation, approximating ellipsoid, guaranteed state estimation, a robust algorithm, a criterion of observations information value, a step of algorithm, Kalman filter.*

1. INTRODUCTION

In this work the robust algorithm of the guaranteed estimation of a set of possible states of the linear controlled system is developed and investigated [1]. In this article we mean conformity of the algorithm with two conditions by the robustness property: first, conservation of operability at the violation of prior guesses of the observable object parameters; secondly, “non-deterioration” of the state estimate of the object during estimation. For this purpose in the algorithm intermediate and final results are checked against prior guesses and a current estimate of the system state. At the violation of prior guesses or estimate deterioration the algorithm parameters are redefined. A result of work of the algorithm can be geometrically represented as an ellipsoid approximating the crossing of a priori ellipsoidal set of attainability of the linear controlled system [1] and a hyperlayer representing according to the data of observation a set of possible states of the controlled system limited by two parallel hyperplanes. In practice such a problem arises, for example, at the accelerated alignment of gyrostabilized platforms under conditions of statistical uncertainty of external influences; correction in the complex orientation and navigation systems [2-7] when an accuracy class of these devices does not enable to achieve marked improvement of quality of work by using data processing complex algorithms.

An approximating ellipsoid minimum volume is taken as a criterion of optimality. This criterion is convenient because of invariance of affine transformations relative to initial sets for obtaining of a minimum ellipsoid [1]. The parameters of the minimum volume ellipsoid circumscribed about a spherical layer or a segment have been evaluated in [8] for a hemisphere and in [9] for a hyperlayer. In [10] we obtain the algorithm of ellipsoidal approximation of the ellipsoid and hyperplane crossing where an approximating ellipsoid volume dependent on several parameters is expressed through the function of these parameters called a step of algorithm. The further works have been devoted to the development of universal and more convenient algorithms for obtaining of the specified approximation. In work [11], for example, for obtaining of the parameters of the minimum volume ellipsoid approximating the ellipsoid and hyperlayer crossing a quadric equation should be solved.

In [12] the algorithm in which a step of algorithm of approximating ellipsoid obtaining has only two values depending on a degree of the hyperlayer and initial ellipsoid crossing has been suggested. Though it greatly simplifies calculations, but a possibility of specification of the system state ellipsoidal estimate considerably decreases, respectively. In [13] we have obtained a condition of step choice for construction of the approximating ellipsoid when an initial ellipsoid and a hyperlayer only touch each other. Dependence of improvement of the system state estimate on a relative width of the hyperlayer has been shown.

1. PROBLEM DEFINITION

E_j system state ellipsoid is set as

$$\{x_j : (x_j - \bar{x}_j)^{\mathrm{T}} H_j^{-1} (x_j - \bar{x}_j) \le 1\} \qquad (1)$$

$x_j \in E_j$, $j \in T_0$, $j = 1, ..., k$, $(k < \infty)$ is discrete time; $E_j \subset X_j = R^n$ is a compact set of possible values of the initial state, $\bar{x}_j$ and $H_j^{\mathrm{T}} = H_j > 0$ are set n is a measuring vector and $(n \times n)$ is a matrix, respectively. The E_j change in the course of time is determined with dynamic system properties [13].

Watch equation

$$y_j = h^{\mathrm{T}} x_j + \xi_j, \left|\xi_j\right| \le c, \ j = 1, 2, ..., \qquad (2)$$

where $y_j \in R^1$; $h \in R^n$, $\|h\| = 1$ is a measuring device parameter; $\xi_j \in R^1$ is a restricted measurements interference; $c \ge 0$ is a set constant. The equation (2) in R^n space determines the hyperlayer

$$S_j = \{x_j : \ (y_j - h^{\mathrm{T}} x_j)^2 \le c^2\}. \qquad (3)$$

On the basis (1) and (3) according to work [13] the guaranteed esmimate $E_{j+1} \supset E_j \cap S_j$

$$E_{j+1} : \ (x_{j+1} - \bar{x}_{j+1})^{\mathrm{T}} H_{j+1}^{-1} (x_{j+1} - \bar{x}_{j+1}) \le 1 \qquad (4)$$

less by volume than the initial ellipsoid (1) is built, otherwise the initial ellipsoid remains.

$$\tilde{x}_{j+1} = \tilde{x}_j + \tau_j \frac{H_j h_j}{e_j} \sigma_j \qquad (5)$$

$$H_{j+1} = (H_j - \tau_j \frac{H_j h h^{\mathrm{T}} H_j}{e_j^2}) \gamma_j^2 \qquad (6)$$

$$\gamma_j^2 = 1 + \tau_j \left(\frac{1}{1 - \tau_j} \chi_j^2 - \sigma_j^2 \right) \qquad (7)$$

$$\tau_j \le \frac{e_j^2}{q_j^{-1} + e_j^2}, \ 0 \le \tau_j < 1, \ q_j^{-1} > 0 \qquad (8)$$

Here:

$e_j^2 = h_j^{\mathrm{T}} H_j h_j$; $\sigma_j = \frac{\Delta_j}{e_j}$; $\chi_j^2 = \frac{c^2}{e_j^2}$;

$\Delta_j = y_j - h_j^{\mathrm{T}} \bar{x}_j$ is a distance from the centre of the initial ellipsoid to the hyperlayer middle;

$h_j \in R^n$ is a parameter of the observer; $\bar{x}_j$ is a centre of the initial ellipsoid;

$\bar{x}_{j+1}$ is a centre of the approximating ellipsoid;

If a condition of a priori ellipsoid and observation compatibility is violated: $1 + \chi \ge \sigma$ scaling should be performed $H_{j+1} = \sigma_j^2 H_j$.

A condition of observations informativity [12]:

$$\frac{\det H_{j+1}}{\det H_j} = (1 - \tau_j) \left(1 + \tau_j \left(\frac{\chi_j^2}{1 - \tau_j} - \sigma_j^2 \right) \right)^n \le 1. \ (9)$$

Fig. 1 for case $n = 2$ shows the crossing of E_j ellipsoid and S_j observations hyperlayer, and approximation of the crossing of abscissas by the ellipsoid E_{j+1}.

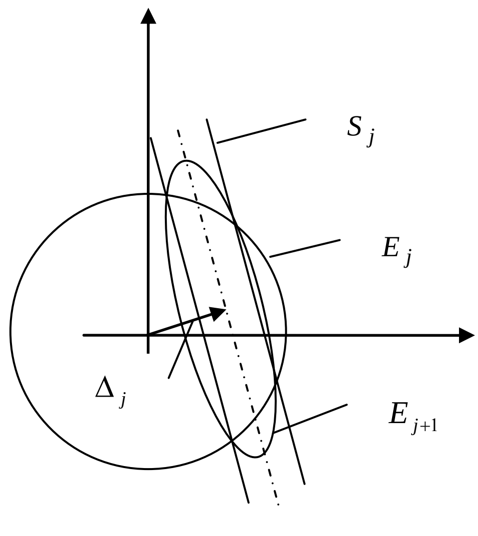

Fig. 1 – Ellipsoidal approximation of ellipsoid and hyperlayer crossing

2. APPROXIMATION OF CROSSING OF THE ELLIPSOIDAL SET OF ATTAINABILITY AND HYPERLAYER BY THE ELLIPSOID

A simplified step is used

$$\tau = \left(1 + n\left(\sigma^2 - \chi^2\right)\right) / 1 + n\sigma^2 \quad (10)$$

$$q = \left(1 + n\left(\sigma^2 - \chi^2\right)\right) / n\chi^2 e^2 . \quad (11)$$

η parameter corresponds q parameter in [11]

$$\eta = \left((1-\sigma^2) - (2n-1)\chi^2 + \sqrt{D}\right) / 2(n-1)\chi^2 e^2 \quad (12)$$

$$D = ((2n-1)\chi^2 - (1-\sigma^2))^2 - 4(n-1)\chi^2 (n(\chi^2 - \sigma^2) - 1)$$

χ_{bound} boundary half-width of the hyperlayer has been determined. The “observation result cancellation” takes palce with it. It is the same for (11) and (12)

$$\chi_{bound} = \sqrt{\left(1 + n\sigma^2\right) / 2n} . \quad (13)$$

Let us assume $|\sigma| + \chi = 1$. Then with $\sigma = (n-1)/2n$ we shall obtain maximum half-width of the hyperlayer

$$\chi = (n+1)/2n \quad (14)$$

Let us assume in expression (13) $\sigma = 0$. Having directed $n \to \infty$ we shall obtain from (13) and (14)

$$\lim_{\sigma=0, n\to\infty} \chi_{bound} = 0, \ \lim_{n\to\infty} \sigma = (n-1)/2n = 1/2,$$

$$\lim_{n\to\infty} \chi_{bound} = (n+1)/2n = 1/2. \quad (15)$$

Fig. 2 and 3 show the diagrams of observations informativity dependence on hyperlayer width (ratios $\det H_{j+1} H_j^{-1}$) for $n = 2$ case. In Fig. 2 the middle of observations hyperlayer passes the centre of the initial ellipsoid, i.e. $\sigma = 0$. In Fig. 3 – $\sigma = (n-1)/2n$. The hyperlayer width χ is set by the abscissa axis, and by the ordinate axis – a value of informativity criterion (9). The full line corresponds to a step by formula (11), the dash line – to (12).

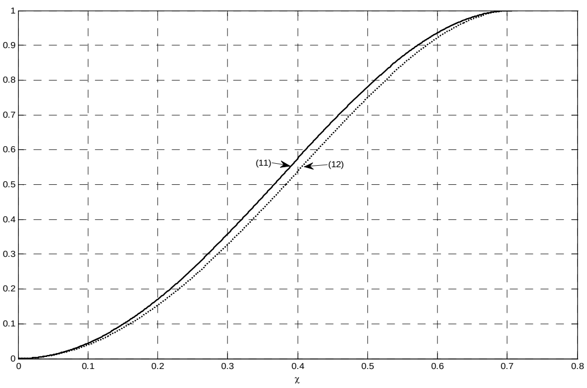

Fig. 2 – Observations informativity change depending on $\sigma = 0$ hyperlayer width

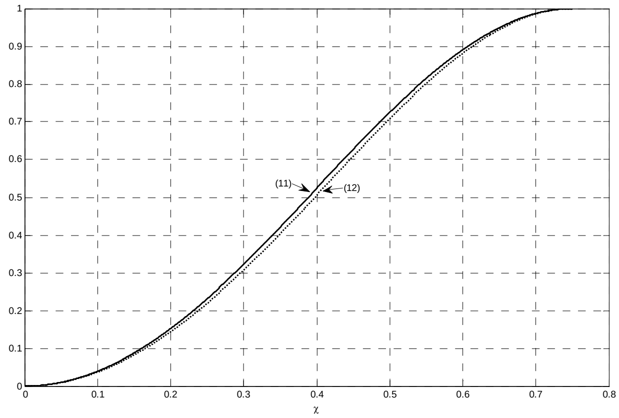

Fig. 3 – Observations informativity change depending on $\sigma = (n-1)/2n$ hyperlayer width

It follows from (15) that with n growth the requirements for accuracy of the observer grow as well.

The observable system states ellipsoid parameters (including an ellipsoid volume) will be connected with the measuring device parameters, measurement interference, system properties and external disturbance by means of the following dependence

$$h^{\mathrm{T}} H h \geq c^2 n \quad (16)$$

If h vector is an eigenvector or close to such a vector of the matrix corresponding to the least eigenvalue of H [16] matrix, we shall have a small volume of the approximating ellipsoid with small uncertainty by one phase coordinate corresponding to the least eigenvalue, and with greater uncertainty by other phase coordinates.

3. MODELLING

There is a linear controlled system

$$x_{j+1} = Ax_j + Bu_j + L_n \zeta_j, \ \left|\zeta_j\right| \leq d, \ x_0 \in E_0, \quad (17)$$

$$E_0 = \{x_0 : (x_0 - \bar{x}_0)^{\mathrm{T}} \bar{H}_0^{-1} (x_0 - \bar{x}_0) \leq 1\}. \quad (18)$$

where A – $(n \times n)$ is a matrix; L_n and B – n are measuring vectors. The pair (A, B) is controlled [15]; $\zeta_j \in R^1$ is a scalar disturbance restricted with $d \geq 0$ set constatnt; $\bar{x}_0$ и $\bar{H}_0^{\mathrm{T}} = \bar{H}_0 > 0$ known n is a measuring vector and $(n \times n)$ is a matrix. Controls $u_j \in R^1$ are set at the whole interval T

$$\{u_j \in R^1, j \in T\}, \ \bar{x}_j = A\bar{x}_{j-1} + Bu_{j-1} \quad (19)$$

$$E_j = \{x_j : (x_j - \bar{x}_j)^{\mathrm{T}} H_j^{-1} (x_j - \bar{x}_j) \leq 1\} \quad (20)$$

The following parameters are taken for the model:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0{,}1875 & 0{,}25 & 0{,}75 \end{bmatrix}; \qquad H_0 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 90 \end{bmatrix};$$

$$\bar{x}_0^{\mathrm{T}} = [0,\ 0,\ 0];\ x_0^{\mathrm{T}} = [1, -2,\ 2];\ L_n^{\mathrm{T}} = [0, 0, 1];$$

$$B^{\mathrm{T}} = [0, 0, 1];\ h^{\mathrm{T}} = [1, 0, 0];\ u_j = 5,\ c = 0{,}1.$$

Modelling has been carried out in MATLAB according to [13]. In the process of estimation a discontinuous change of the system dynamic properties by means of replacement of the 3d line of A matrix to $a_3 = [0{,}26 \quad -0{,}92 \quad 1{,}3]$ has been simulated. The disturbing influence bound has been increased at the same time: $d = 2$. And the algorithm has operated with A initial matrix and bound $d = 1$.

Fig. 4 shows a membership function value without change of observable system dynamics, but with scaled-up disturbance bound. The full line is the changed observable system, the dash line is unchanged system.

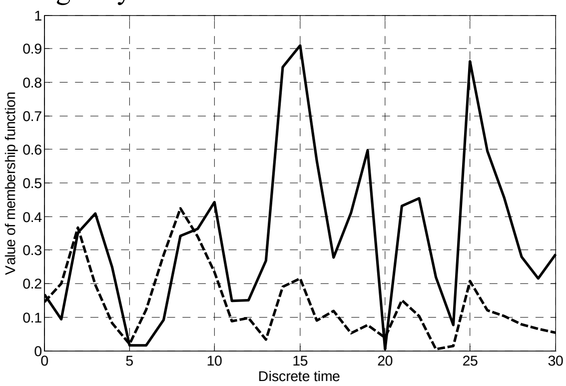

Fig. 4 – Membership function – check of fulfillment of conditions (20) for the observable system with a scaled-up disturbance bound

Fig. 5 shows a membership function value with changed dynamics of the observable system and scaled-up disturbance bound. The full line is a membership function for the changed observable system, the dash line – for the unchanged system.

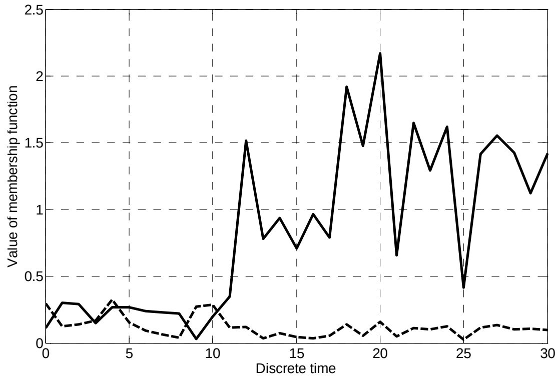

Fig. 5 – Membership function – check of fulfillment of conditions (20) for the system with a scaled-up disturbance bound and changed dynamics

4. CONCLUSION

By the results of algorithm operation modelling and investigations of dependence between its parameters and parameters of the observable system such algorithms can be recommended for application in case of small spatial dimension of system states. For the same reason possible algorithm modifications shall be checked on deterioration of observations informativity i.e. on decrease in sensitivity to useful signal extraction from the observer “noise”.

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