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ROBUST STABILITY AND EVALUATION OF THE QUALITY FUNCTIONAL OF LINEAR DISCRETE SYSTEMS WITH MATRIX UNCERTAINTY

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Consider a linear dynamical control system with discrete time which describing difference equations in the form:

$$x_{t+1} = (A + \Delta A_t)x_t + (B + \Delta B_t)u_t, \quad y_t = Cx_t + Du_t, \tag{1}$$

where $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^m$ and $y_t \in \mathbb{R}^l$ are state, control, and observable object output vectors respectively, $t = 0, 1, 2, ...$, A, B, C and D are constant matrices of corresponding sizes $n \times n$, $n \times m$ and $l \times n$, $l \times m$, and

$$\Delta A_t = F_A \Delta_{At} H_A, \quad \Delta B_t = F_B \Delta_{Bt} H_B,$$

where F_A, F_B, H_A, H_B — are constant matrices of corresponding sizes and matrices uncertainties Δ_{At} and Δ_{Bt} satisfy the constraints $\|\Delta_{At}\| \le 1, \|\Delta_{Bt}\| \le 1$ or $\|\Delta_{At}\|_F \le 1, \|\Delta_{Bt}\|_F \le 1$, $t = 0, 1, 2, ...$ $\|\cdot\|$ is Euclidean vector norm and spectral matrix norm, $\|\cdot\|_F$ is matrix Frobenius norm.

We control the system (1) with output feedback:

$$u_t = Ky_t, \quad K = K_0 + \widetilde{K}, \quad \widetilde{K} \in \mathcal{E} = \{K : K^T P K \le Q\}, \tag{2}$$

where $P = P^T > 0$ and $Q = Q^T > 0$ are symmetric positive definite matrices.

Consider a control system (1), (2) with quadratic quality functional

$$J_u(x_0) = \sum_{t=0}^{\infty} \varphi_t, \ \varphi_t = \begin{bmatrix} x_t^T & u_t^T \end{bmatrix} \Phi \begin{bmatrix} x_t \\ u_t \end{bmatrix}, \ \Phi = \begin{bmatrix} S & N \\ N^T & R \end{bmatrix} > 0,$$

where x_0 is initial vector, $S = S^T > 0$, $R = R^T > 0$ and N given constant matrices.

We introduce on the set of matrices $\mathcal{K} = \{K : \det(I_m - KD) \ne 0\}$ a nonlinear operator

$$\mathcal{D} : \mathbb{R}^{m\times l} \to \mathbb{R}^{m\times l}, \quad \mathcal{D}(K) = (I_m - KD)^{-1}K \equiv K(I_l - DK)^{-1}.$$

Теорема. Suppose that for a positive definite matrix $X = X^T > 0$ and for some $\varepsilon_i > 0$ $(i = 1, 2, 3)$ the following matrix inequalities hold:

$$\begin{bmatrix} R - G^T P G + \varepsilon_1^{-1} H_B^T H_B & D^T & B^T \\ D & -Q^{-1} & 0, \\ B & 0 & -X^{-1} + \varepsilon_1 F_B F_B^T \end{bmatrix} < 0,$$

$$\begin{bmatrix} -X + \Omega & N_*^T & C_0^T & M_*^T \\ N_* & R - G^T P G + \varepsilon_3^{-1} H_B^T H_B & D^T & B^T \\ C_0 & D & -Q^{-1} & 0 \\ M_* & B & 0 & -X^{-1} + \Theta \end{bmatrix} < 0,$$

where $\Omega = L_0^T \Phi L_0 + \varepsilon_2^{-1} H_A^T H_A + \varepsilon_3^{-1} C_*^T C_*$, $\Theta = \varepsilon_2 F_A F_A^T + \varepsilon_3 F_B F_B^T$, $M_* = A + B\mathcal{D}(K_0)C$, $N_* = N^T + R\mathcal{D}(K_0)C + \varepsilon_3^{-1} H_B^T C_*$, $C_* = H_B \mathcal{D}(K_0)C$, $L_0^T = \begin{bmatrix} I_n & C^T \mathcal{D}^T(K_0) \end{bmatrix}$, $C_0 = C + D\mathcal{D}(K_0)C$, $G = I_m - K_0 D$. Then any control (2) ensures asymptotic stability of the zero state for system (1), the general Lyapunov function $v(x_t) = x_t^T X x_t$, and a bound on the functional $J_u(x_0) \le \omega$.