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# The Method of Identification of a Mathematical Model for the Cardiovascular System Response Dynamics to Exercise Stress

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**Abstract.** *The process of rehabilitation after heart disease is a major problem today. Existing mathematical model is phenomenological. This paper presents a mathematical model of the heart rate and pressure under the influence of physical activity. The identification of the presented model based on the modified gradient Levenberg-Markvadt method supplemented with the procedure of initial ratios choice is introduce for the first time.*
**Keywords:** *Mathematical model, cardiovascular system, identification, exercise stress, heart rate, blood pressure.*

## 1. Introduction

One of today's applied problems which appear in medicine is to predict the heart rate and blood pressure dynamics under exercise stress. This is the main task in the planning process of rehabilitation after the cardiovascular system diseases, especially myocardial infarction. Today it is dealt with using empirical methods; however, the use of mathematical modeling would allow avoiding subjective assessments and increase the reliability of the forecast.

The work is dedicated to building a model of the pulse and pressure dynamics under the influence of physical activity and the method of their identification. The researches that are devoted to the mathematical modeling of the cardiovascular and respiratory systems of the human body under exercise stress have been analyzed to find a theoretical basis for this study.

On the basis of similarities between the cardiovascular system of man and electrical systems O. Frank in the early 20th century suggested modeling the cardiovascular system through an electrical circuit [1]. This idea is implemented in a series of papers [2]-[7]. The disadvantages of this approach are the relatively low accuracy and complexity of construction, of identification and modification of the model by a professional not familiar with the theory of electrical circuits. Representation of the cardiovascular system with a system of differential equations acts as much more powerful tool for modelling.

Well-known paper [8] is based on the use of differential equations that model the work of a four-chambered heart described by Grodins and characterizes the work of small and large circles of blood circulation, left and right ventricles and baroreceptors. Also, this model takes into account Starling's and Bowditch's effects and self-regulation in the peripheral areas. This model can be used to analyze the blood pressure in a brain and measure blood pressure during orthostatic stress. In [9]-[10], based on the study [8], there is formed the overall picture of combination of the cardiovascular and respiratory systems in the form of eleven differential equations representing contractility of the left and right ventricles of the heart, the relationship between heart rate and contractility, gases balance equation, oxygen consumption in the exercise process, the exchange of gases in the tissue of the body. In [11] the models of the cardiovascular and respiratory systems are combined with a quantitative representation of the executable exercise.

Models, which are described in [3]-[11], are phenomenological models that show the flow of processes in the body at a qualitative level. At the same time, the practical use of models of the cardiovascular system must take into account the specific features of the body and we should have the possibilities of creating a database of statistical information based on the data used in daily clinical practice, i.e. based only on the dynamics of the heart rate and blood pressure. Thus following the ideology of [12]-[13] we construct a mathematical model of the response of the cardiovascular system to exercise stress, which will manifest itself in dynamics of heart rate and blood pressure, and provide an identification method for the developed model.

## 2. Mathematical Model

Clinical experience has identified two main stages of functioning of the cardiovascular system under exercise stress: period of response of the cardiovascular system to stress and recovery period. The last one is accompanied with increased values of pulse and pressure and they increase in proportion to the intensity of physical activity, while the recovery period makes them return to their original state. The general structure of the model is visualized in Figure 1
$W$ is load, $W'$ is change of load, $H, P$ are heart rate and blood pressure, $H_0, P_0$ are

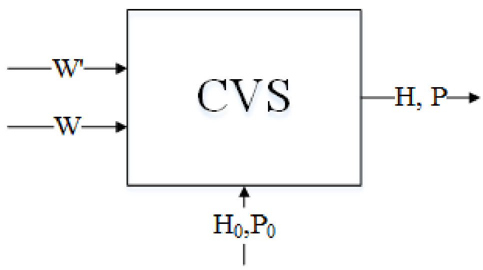



Figure 1. Parameters which characterize cardiovascular system (CVS) under the influence of physical activity to improve heart rate and pressure

initial values of heart rate and blood pressure.

The stage of the body's response to exercise can be selected using the Michaelis-Menten function:

$$M(t) = \frac{M(t)}{1 + M(t)}, \tag{1}$$

which is a smooth analogue of the Heaviside function. Its value is close to 1 at high volume load and reduced to 0 with a sharp decrease in physical activity. The last activates the body's recovery process defined as characteristic equal to $1 - M(t)$. The period of the phase transition of the organism from stress to recovery is characterized by certain inertia. It is shown as the delay argument $t - t_0$ in the characteristics of the recovery process

$$R(t) = 1 - M(t - t_0) = 1 - \frac{W(t - t_0)}{1 + W(t - t_0)}, \tag{2}$$

where $t_0$ is transition duration from the time of unloading until the activation process of recovery.

As the pressure changes are proportional to the change of physical activity as $p, h \approx W'$, their dynamics can be described by the following Cauchy problem for a set of differential equations:

$$h'(t) = A_1 W'(t)\frac{W(t)}{(1 + W(t - t_0))} - \left(1 - \frac{W(t - t_0)}{(1 + W(t - t_0))}\right)A_2 h^{A_3}(t), \tag{3}$$

$$p'(t) = B_1 W'(t)\frac{W(t)}{(1 + W(t - t_0))} - \left(1 - \frac{W(t - t_0)}{(1 + W(t - t_0))}\right)B_2 h^{B_3}(t), \tag{4}$$

$$h(0) = 0, \tag{5}$$

$$p(0) = 0, \tag{6}$$

where $A_1, B_1$ are the indicators of the dynamics of the exercise stress on the change of heart rate and blood pressure; $A_2, B_2$ are the indicators of speed adaptation to exercise stress relieving; $A_3, B_3$ are the coefficients of the heart rate and blood pressure influence in the process of adaptation to unloading; $h, p$ are excess levels of functional heart rate $H_m$ and blood pressure $P_m$.

$$h = H - H_m, \tag{7}$$

$$p = P - P_m. \tag{8}$$

## 3. Identification of Mathematical Model

The offered mathematical model requires parameter identification, which can be carried out by the root-mean-square criterion.

$$\vec{a} = arg \min_{\vec{A}} \sum_{j=1}^{N_t} (\tilde{h}(\vec{A}, t_j) - (H_j - H_m))^2, \tag{9}$$

$$\vec{b} = arg \min_{\vec{B}} \sum_{j=1}^{N_t} (\tilde{p}(\vec{B}, t_j) - (P_j - p_m))^2, \tag{10}$$

where $N_t$ is dimension of the set of points measured for the experiment; $\vec{a}, \vec{b}$ is the solution of differential equations for a given set of coefficients $\vec{A}, \vec{B}$ at the observation point $t$.

To minimize the functional (9)–(10) let's use modified Levenberg–Marquardt gradient method. Its implementation needs to build the initial approximation for the coefficients of the model. The identification algorithm is divided into two levels. The top level is designed to search the optimal values of coefficients $A_3, B_3$ using the exhaustive search of possible values on a uniform grid covering some of empirically selected ranges. After selecting coefficients $A_3, B_3$, coefficients $A_1, A_2$, $B_1, B_2$ are determined by the Levenberg-Marquardt method. Their initial values are set by the following equations:

$$A_1 = \frac{h^{'}}{W'} = \frac{h_3 - h_1}{W_3 - W_1}, \tag{11}$$

$$A_2 = \frac{h_k - h_{k+2}}{2\Delta t \cdot h_k} \tag{12}$$

$$B_1 = \frac{p^{'}}{W'} = \frac{p_3 - p_1}{W_3 - W_1}, \tag{13}$$

$$B_2 = \frac{p_k - p_{k+2}}{2\Delta t \cdot p_k} \tag{14}$$

where $t$ is time of exercise stress removal.

Considering that $W(t) \approx 1, R(t - t_0) \approx 0, W(t_k) \approx 0, R(t - t_0) \approx 1$ are at the start of the reaction process and dynamics change parameters are close to linear at the beginning of the recovery process. In this case the differential equations could be simplified and the corresponding values are obtained from the approximation of difference-differential operators ratios.

# 4. Results

To check the adequacy of the designed model we have used a tool for checking exercise capacity, i.e. ergometer. The observations were conducted over a group of patients of the lightest functional class who were at the final stage of the rehabilitation process. Dynamics of reactions to an identical exercise was recorded at intervals of 10 days during the process of the final stage of the rehabilitation program. The values of evolution of the dynamics parameters of the cardiovascular system for some of the observed patients are presented in Table 1.

The data were used to simulate the heart rate and blood pressure at various stages of rehabilitation using the assessment of maximum relative errors. The values of the identified errors and ratios are set forth in Table 2.

Table 1. The values of the heart rate and blood pressure received during the inspection for exercise capacity

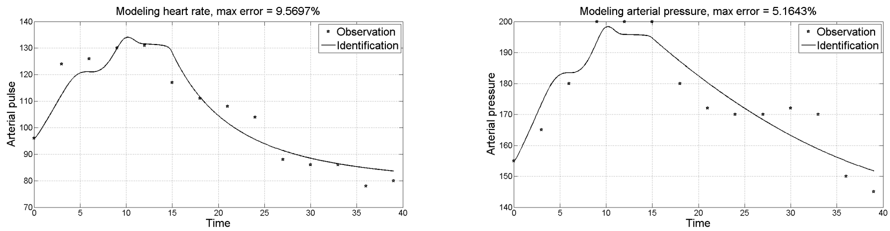
| **W** | **H1** | **H2** | **H3** | **H4** | **H5** | **P1** | **P2** | **P3** | **P4** | **P5** |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 96 | 94 | 93 | 82 | 61 | 158 | 150 | 155 | 140 | 130 |
| 25 | 124 | 115 | 115 | 90 | 81 | 175 | 165 | 170 | 165 | 150 |
| 25 | 120 | 110 | 119 | 88 | 82 | 180 | 175 | 170 | 165 | 150 |
| 50 | 130 | 120 | 125 | 90 | 87 | 200 | 180 | 180 | 180 | 160 |
| 0 | 131 | 122 | 119 | 83 | 68 | 200 | 178 | 170 | 180 | 170 |
| 0 | 117 | 110 | 97 | 75 | 61 | 200 | 175 | 170 | 170 | 172 |
| 0 | 111 | 95 | 89 | 70 | 64 | 180 | 172 | 165 | 170 | 165 |
| 0 | 108 | 92 | 88 | 70 | 60 | 172 | 170 | 165 | 160 | 140 |
| 0 | 104 | 97 | 85 | 71 | 63 | 170 | 165 | 160 | 160 | 138 |
| 0 | 88 | 95 | 82 | 72 | 63 | 170 | 164 | 160 | 155 | 134 |
| 0 | 86 | 91 | 87 | 77 | 62 | 172 | 160 | 156 | 158 | 140 |
| 0 | 86 | 92 | 85 | 74 | 62 | 170 | 145 | 156 | 155 | 135 |
| 0 | 78 | 88 | 81 | 72 | 63 | 150 | 141 | 155 | 150 | 135 |
| 0 | 80 | 88 | 79 | 73 | 62 | 145 | 140 | 150 | 140 | 130 |

Table 2. The value of heart rate and blood pressure received during the inspection body for exercise capacity

| **N** | $\mathbf{x_1}$ | $\mathbf{x_2}$ | $\mathbf{x_3}$ | $\mathbf{x_4}$ | **ErrH**, % | **ErrP**, % |
|---|---|---|---|---|---|---|
| 1 | 0,7656 | 0,0303 | 0,8456 | 0,3704 | 9,53 | 5,16 |
| 2 | 0,6250 | 0,0616 | 0,6426 | 0,3519 | 3,33 | 4,14 |
| 3 | 1,4528 | 0,0471 | 0,4092 | 0,0899 | 2,00 | 2,75 |
| 4 | 0,6027 | 0,0160 | 0,7779 | 0,0701 | 7,92 | 4,93 |
| 5 | 0,9291 | -0,0056 | 0,7303 | 0,2704 | 6,09 | 3,93 |

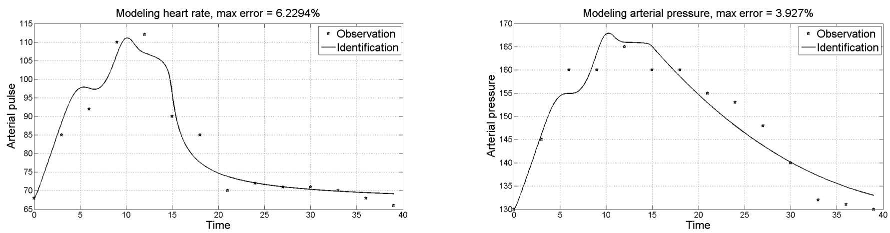
The maximum error during the whole period was 9,53% for heart rate and 5,16% for the blood pressure.

The obtained results indicate the adequacy of the proposed mathematical model for the analyzed phase of the rehabilitation process. The results of the identification of the model describing dependence of the main organism stress indicators at the initial and final periods of rehabilitation process are visually presented in Figures 2-3.




(a) heart rate (b) arterial pressure

Figure 2. The results of identification of the dynamic model parameters of the heart rate and blood pressure under exercise stress of the initial phase




(a) heart rate (b) arterial pressure

Figure 3. The results of identification of the dynamic model parameters of the heart rate and blood pressure under exercise stress of the final stage

Analysis of the graphs shows a decline of initial values of the heart rate and blood pressure as a result of the rehabilitation program; there is also acceleration of the process of recovery of the heart rate when exercise stress is removed.

# 5. Conclusions

This paper presents peculiarities of the process of rehabilitation after uncomplicated myocardial infarction, the analysis of the main approaches with emphasis on their main weaknesses. A mathematical model of the heart rate and blood pressure under the influence of physical activity is designed using a set of differential equations taking into account the Michaelis-Menten law during the period of rehabilitation, which allows predicting the body's response to dosed physical load. The method of identifying a set of differential equations that model the dynamics of heart rate and blood pressure under the influence of physical activity on the basis of the modified gradient Levenberg-Markvadt method supplemented with the procedure of initial ratios choice is introduced for the first time and that made possible to confirm the adequacy of the developed mathematical model.

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