6th Ya.B. Lopatynsky International School-Workshop on Differential Equations and Applications

June 18-20, 2019
Vinnytsia, Ukraine

Book of Abstracts

Ministry of Education and Science of Ukraine
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LECTURES OF MAIN SPEAKERS

SHORT COMMUNICATIONS

Classical solution to the Poisson's equation

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Consider the Poisson's equation [1, c. 276]

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = -f(x,y), \tag{1}$$

and such function:

$$u_1(x,y) = \frac{i}{2}\int_0^y d\eta \int_{x-i(y-\eta)}^{x+i(y-\eta)} f(\xi,\eta)d\xi, \quad i=\sqrt{-1}. \tag{2}$$

Find the partial derivatives of the first and second orders of the function $u_1(x,y)$. On the basis of formula (2) we find

$$\frac{\partial u_1}{\partial x} = \frac{i}{2}\int_0^y \left(f(x+i(y-\eta),\eta) - f(x-i(y-\eta),\eta)\right) d\eta. \tag{3}$$

Introduce the notation: $\alpha(x,y,i,\eta) = x + i(y-\eta)$; $\beta(x,y,i,\eta) = x - i(y-\eta)$. For $f(x,y) \in C^{1,0}$ we have

$$\frac{\partial^2 u_1}{\partial x^2} = \frac{i}{2}\int_0^y \left(\frac{\partial f(\alpha(x,y,i,\eta),\eta)}{\partial \alpha} - \frac{\partial f(\beta(x,y,i,\eta),\eta)}{\partial \beta}\right) d\eta;$$

$$\frac{\partial u_1}{\partial y} = \frac{i}{2}\int_0^y \left(if(x+i(y-\eta),\eta) + if(x-i(y-\eta),\eta)\right) d\eta;$$

$$\frac{\partial^2 u_1}{\partial y^2} = \frac{i}{2}\int_0^y \left(i^2\, \partial f(\alpha(x,y,i,\eta),\eta) - i^2\, \partial f(\beta(x,y,i,\eta),\eta)\right) d\eta - f(x,y).$$

So,

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = -f(x,y).$$

This signifies that the function $u_1(x,y)$ defined by the formula (3) is a partial solution to the equation (1).

Similarly we prove, the function

$$u_2(x,y) = \frac{i}{2}\int_y^\pi d\eta \int_{x+i(y-\eta)}^{x-i(y-\eta)} f(\xi,\eta)d\xi, \quad i=\sqrt{-1}, \tag{4}$$

for $f(x,y) \in C^{1,0}$, is a partial solution to the equation (1).

Theorem. If the function $f(x,y) \in C^{1,0}$, then the function

$$u(x,y) = \frac{i}{4}\int_0^y d\eta \int_{x-i(y-\eta)}^{x+i(y-\eta)} f(\xi,\eta)d\xi + \frac{i}{4}\int_y^\pi d\eta \int_{x+i(y-\eta)}^{x-i(y-\eta)} f(\xi,\eta)d\xi.$$

is the classical ($u \in C^{2,2}$) solution to the equation (1).

References

[1] A. N. Tikhonov, A. A. Samarskiy, Equations of mathematical physics, M Nauka, 1977, 735 p.

Operator research of classical solutions to boundary-value problems for hyperbolic second or equations

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Statement of the problem: to find a classical solution to the hyperbolic second equation

$$u_{tt} - u_{xx} = g(x,t), \quad 0 \le x \le \pi, \quad t \in \mathbb{R},$$

which satisfies the boundary conditions

$$u(0,t) = u(\pi,t) = 0, \quad t \in \mathbb{R}.$$

We have proved that the classical solution to the problem (1), (2) is the function [1,

$$u(x,t) = (Rg)(x,t) \equiv (Sg)(x,t) + \left(\widetilde{S}g\right)(x,t),$$

where

$$(Sg)(x,t) = -\frac{1}{4}\int\limits_0^x d\xi \int\limits_{t-x+\xi}^{t+x-\xi} g(\xi,\tau)\,d\tau - \frac{1}{4}\int\limits_x^\pi d\xi \int\limits_{t+x-\xi}^{t-x+\xi} g(\xi,\tau)\,d\tau;$$

$$\left(\widetilde{S}g\right)(x,t) = \frac{\pi - x}{4\pi}\int\limits_0^\pi d\xi \int\limits_{t-\xi}^{t+\xi} g(\xi,\tau)\,d\tau - \frac{x}{4\pi}\int\limits_0^\pi d\xi \int\limits_{t-\pi+\xi}^{t+\pi-\xi} g(\xi,\tau)\,d\tau;$$

$g(x,t) \in C^{0,1}$, $\quad C^{0,1}-$ is the space of functions of two variables, continuous and boun together with the derivative of t, defined on the set $[0,\pi] \times \mathbb{R}$.